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A covering second-order Lagrangian for the relativistic top without forces. (English summary)

Symmetry in nonlinear mathematical physics, Part 1, 2 (Kyiv, 2001), 741–745, *Pr. Inst. Mat. Nats. Akad. Nauk Ukr. Mat. Zastos.*, 43, *Part* 1, 2, *Natsīonal. Akad. Nauk Ukraïni, Īnst. Mat.*, *Kiev*, 2002.

In this interesting paper a Lagrangian (of second order) and a Hamiltonian (Ostrogradskiĭ) for a free massive relativistic spinning top are presented. The relativistic top is described in terms of its linear four-momentum and its four-velocity which are not parallel to each other. Consequently the derivative of the spin degrees of freedom is expressed in terms of the four-momentum and the four-velocity. The author therefore starts with the Dixon equations in an arbitrary pseudo-Riemannian space-time augmented by the extra condition (Pirani constraint) stating that the four-velocity contracted with the spin tensor should be zero during the motion. Using this constraint the author is able to eliminate the four-momentum variable from the equations of motion completely. Restricting the motion to the flat space-time, a further elimination of the spin degrees of freedom leads to a nontrivial fourth order differential equation for the position variable of the top. Using the proper time as the parameter the equation says that the sum of the fourth derivative (with respect to the proper time) of the position of the free relativistic top plus a positive constant times the second derivative (with respect to the proper time) of the position equals zero. The solutions correspond in this way (loosely speaking) to a "zitterbewegung" motion of the free relativistic spinning top. Next, the author shows how the so-obtained equations of motion (now with respect to an arbitrary parameterization of the position variable of the relativistic spinning top) can be given a second order Lagrangian formulation and further a Hamiltonian-Ostrogradskiĭ formulation. In the Hamiltonian on page 743 (as pointed out by the author) in the last line there is a minor error. The last term in the Hamiltonian should read instead $\frac{-A}{2}\sqrt{1+v^2}$.

It would be interesting to see what the generators of the Poincaré group look like in terms of the canonical variables introduced by the author and their relations to the original linear four-momentum and spin four-tensor variables.

This and other papers concerned with the motion of relativistic spinning bodies at classical and also at quantum level show that, in spite of the many years passed since the subject was first addressed by scientists, full understanding of and consensus about this topic has not yet been achieved.

{For the entire collection see MR1898973 (2003b:00031)}

Andreas Bette