MR1686048 (2000b:58031) 58E30 58A20
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Integration by parts and vector differential forms in higher order variational calculus on fibred manifolds. (English, Russian summaries)

Mat. Stud. **11** (1999), *no. 1,* 85–107.

The basic goal of this paper is to provide a first variation formula for higher-order Lagrangian densities. Let $\pi_r\colon Y_r \to Z$ be the r-jet bundle over Z of local sections of a fibered manifold $\pi\colon Y \to Z$ with projections ${}^r\pi_s\colon Y_s \to Y_r$, $r < s$, and let V_r be the vertical bundle of π_r, with $V_0 = V$. The basic results are as follows: (1) Given a Lagrangian λ of order r on π, there exist semi-basic differential forms ε, κ, on Y_{2r}, Y_{2r-1}, of degrees p, $p-1$, $p = \dim Z$, with values in V^*, V^*_{r-1}, respectively, such that $({}^r\pi_{2r})^* d_\pi\lambda = ({}^0\pi_r)^\# \varepsilon + d_t\kappa$, where d_π, d_t denote the fiber differential and the total differential, respectively, and the superscript $\#$ means the embedding (with respect to the projection $T({}^0\pi_r)$) of the corresponding modules of vector bundle-valued differential forms. Moreover, ε is unique and κ may be determined, under some additional restrictions, up to a d_t-exact form. (2) The variational derivative of the action density $\check\lambda(\upsilon) = (j_r\upsilon)^*\lambda$ at a section υ of π is a differential operator $\mathbf{D}\check\lambda(\upsilon)$ in the space of variations of υ. If $\mathbf{G}$ denotes the Green operator for $\mathbf{D}\check\lambda(\upsilon)$, then for every variation $\mathfrak{y}$ of υ one has: $(\mathbf{D}\check\lambda)(\upsilon)(\mathfrak{y}) = \langle j_r\mathfrak{y}, (j_r\upsilon)^* d_\pi\lambda\rangle$; ${}^t(\mathbf{D}\check\lambda)(\upsilon)(1) = (j_{2r}\upsilon)^*\varepsilon$; $\mathbf{G}(\mathfrak{y})(1) = \langle j_{r-1}\mathfrak{y}, (j_{2r-1}\upsilon)^*\kappa\rangle$. The operator $\mathbf{G}$ is known to be defined up to a d-closed term. The Euler-Lagrange equations arise as a local expression for the integral sub-manifolds of the vector bundle-valued exterior differential system $(j_{2r}(\upsilon))^*\varepsilon = 0$. *Jaime Muñoz Masqué*