MR634648 (84g:58039) 58E30 70H35
Matsyuk, R. Ya.
On the existence of a Lagrangian for a system of ordinary differential equations. (Russian)
Mat. Metody i Fiz.-Mekh. Polya No. 13 (1981), 34–38, 113.

The author presents the following result. A system of ordinary differential equations $\lambda_j(x^i(\tau), x^{(0)i}(\tau), \cdots, x^{(r)i}(\tau)) = 0$, where $i, j = 1, 2, \cdots, N$, and $x^{(k)i} = d^k x^i/d\tau^k$, is the system of Euler-Poisson equations of a Lagrangian if and only if

$$\frac{\partial\lambda_i}{\partial x^{(0)j}} - \frac{\partial\lambda_j}{\partial x^{(0)i}} - \sum_{s=0}^{r}(-1)^s \frac{d^s}{d\tau^s}\frac{\partial\lambda_j}{\partial x^{(s)i}} - \frac{\partial\lambda_i}{\partial x^{(s)j}} = 0,$$

$$\frac{\partial\lambda_i}{\partial x^{(v)j}} - \sum_{s=v}^{r}(-1)^s \frac{s!}{(s-v)!v!}\frac{d^{s-v}}{d\tau^{s-v}}\frac{\partial\lambda_j}{\partial x^{(s)i}} = 0, \quad 1 \leq v \leq r.$$

The proof is based on W. M. Tulczyjew's results on the structure of the Lagrange differential [C. R. Acad. Sci. Paris Sér. A-B **280** (1975), A1295-A1298; MR0377987 (51 #14156)].

"Analogous general results on systems of partial differential equations can be found in an article by I. M. Anderson and T. Duchamp [Amer. J. Math. **102** (1980), 781–868; MR0590637 (82d:58027)] and a paper by the reviewer [*Proceedings of the conference on differential geometry and its applications* (Nové Město na Moravě, 1980), 181–188, Univ. Karlova, Prague, 1982; MR0663224 (83k:58026); *Proceedings of the IUTAM-ISSIM symposium on modern developments in analytical mechanics* (Torino, 1982), to appear].

Demeter Krupka