MR2553650 (2010i:53065) 53C22 53A04 58E10
Matsyuk, Roman (UKR-AOS-A2)

Second order variational problem and 2-dimensional concircular geometry. (English summary)

Travaux mathématiques. Vol. XVIII, 125–137, *Trav. Math.*, 18, *Fac. Sci. Technol. Commun. Univ. Luxemb.*, *Luxembourg*, 2008.

The author proves the following characterization: Assume that a locally variational third-order autonomous dynamical system $E_i(x^j, u^j, \dot{u}^j, \ddot{u}^j) = 0$ on a two-dimensional manifold possesses (i) Euclidean symmetry, and (ii) the Frenet curvature k as a first integral and all the curves of constant curvature and the straight lines with natural parametrization $\dot{\mathbf{u}} = 0$ as solutions. Then,

$$E_i = \frac{\varepsilon_{ij}\ddot{u}^j}{\|\mathbf{u}\|^3} - 3\frac{\dot{\mathbf{u}}\cdot\mathbf{u}}{\|\mathbf{u}\|^5}\varepsilon_{ij}\dot{u}^j + m\frac{\|\mathbf{u}\|^2\dot{u}_i - (\dot{\mathbf{u}}\cdot\mathbf{u})u_i}{\|\mathbf{u}\|^3}.$$

The expression for the Lagrangian function is given and a variational description of geodesic circles is also obtained.

{For the entire collection see MR2553638 (2010e:00019)}

Jaime Muñoz Masqué