УДК 519.852: 539.3

IMAGING SOLUTIONS OF REACTION-DIFFUSION SYSTEMS USING SAMMON METHOD

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To study the behavior of solutions of dissipative systems vital is the question of approaches to astimating the randomness of solutions and their classification. The Sammon method of data visualization [1] is used to classify different types of solutions of reaction-diffusion systems:

$$\frac{\partial\theta}{\partial t} = l^2\frac{\partial^2\theta}{\partial x^2} + \theta^2 - \eta + 1\,, \quad \frac{\partial\eta}{\partial t} = L^2\frac{\partial^2\eta}{\partial\ x^2} - \eta(\eta - (\theta - A)^3)\,. \qquad (1)$$

Suppose that we have N vectors in an L-space designated X_i, $i=1,.., N$ and respectively we define N vectors in a d-space (d = 2 or 3) designated Y_i, $i=l,..,N$. Let the distance between the vectors X_i and X_j in the L-space be defined by d_{ij}*=$dist\ [X_i,X_j]$ and the distance between the corresponding vectors – Y_i and Y_j in the d-space be defined by $d_{ij}= dist\ [Y_i,\ Y_j]$.

Let us now randomly choose an initial d-space configuration for the Y vectors and denote the configuration as follows: $Y_i = \left[y_{i1},...,y_{id}\right],\ i=1,...,N$.

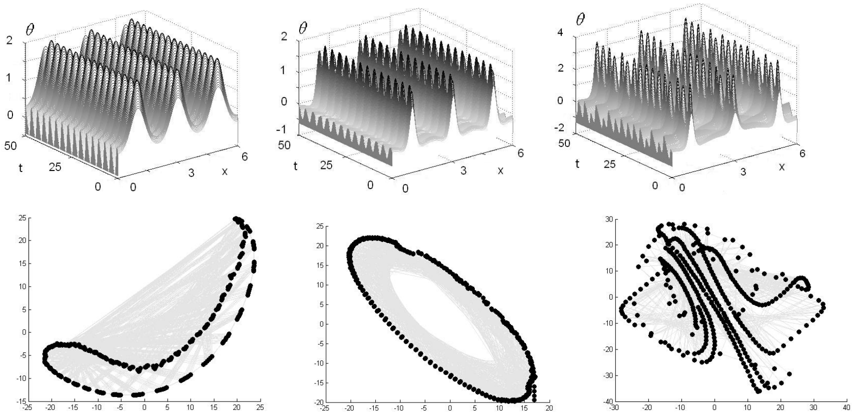
Next we compute all the d-space interpoint distances d_{ij}, which are then used to define an error E, which represents how well the present configuration of N points in the d-space fits the N points in the L-space, i.e.,

$$E = \left(1/\sum_{i<j}\left[d_{ij}^*\right]\right)\sum_{i<j}^{N}\left[d_{ij}^* - d_{ij}\right]^2 / d_{ij}^*\,. \qquad (2)$$

Note that the error is a function of the $d\times N$ variables y_{pq}, $p=1,\ ...\ ,N$ and $q= 1,..,d$. The next step in the Summon algorithm is to adjust the y_{pq} variables or equivalently change the d-space configuration so as to decrease the error. In Sammon method [1] used an iterative optimization Newton method:

$$y_{pq}(k+1) = y_{pq}(k) - \xi\Delta_{pq}(k)\,, \qquad (3)$$

where ξ – coefficient of training (in the interval [0.3, 0.4]), and $\Delta_{pq}(k)$ is the quotient of the corresponding component of the gradient and Hessian diagonal component defined in the k-th iteration: $\Delta_{pg}(k) = \left(\delta E / \delta y_{pq}\right) / \left(\delta^2 E / \delta^2 y_{pq}{}^2\right)$.



a) b) c)

Fig.1 Graphs of the numerical system (1) and Sammon projection if $l^2 = 0.05$, $v = -0.375$ - a) $l^2 = 0.01$, $v = -0.75$ - b) and if $l^2 = 0.01$, $v = -0.375$ - c), $L^2 = 1$.

Sammon mapping on a plane was conducted for different types of solutions of (1) [2]. The result of this mapping is shown in Fig. 1. The first graph shows the projection of data corresponding to regular oscillations, the next one - the projection of data obtained for the bifurcation parameter *A*, close to the critical value when the oscillations lose their regularity, and the last graph corresponds to chaotic oscillations. As one can see, the method is sensitive to changes in the nature of oscillations and makes it possible to clearly delineate the chaotic oscillations from the regular one.

1. *Sammon J. W.* A nonlinear mapping for data structure analysis // IEEE Nrans. On Computers. – 1969. – P. 401-409.
2. *Datsko B.Y., Vasjunyk Z. I.* Classification of solutions to systems of reaction-diffusion based methods for data visualization // Sampling and processing of information. – 2007. – № 26 (102). – P. 114-120 (in ukrainian).

ВІЗУАЛІЗАЦІЯ РОЗВ'ЯЗКІВ СИСТЕМ РЕАКЦІЇ-ДИФУЗІЇ З ДОПОМОГОЮ МЕТОДУ САММОНА

Для дослідження поведінки розв'язків дисипативних систем актуальним є питання підходів до оцінки хаотичності розв'язків та їх класифікації. Метод візуалізації даних Саммона використано для класифікації різних типів розв'язків системи реакції-дифузії, що виявилося достатньо ефективним.