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ON A PRIORI INEQUALITIES FOR THE SYSTEMS DESCRIBED BY PSEUDO-PARABOLIC DIFFERENTIAL EQUATIONS

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The method of a priori inequalities in negative norms is one of the approaches that has been actively used to investigate various issues in mathematical modeling and optimal control, including correctness of the initial-boundary value problem formulation, existence of optimal control, various controllability issues, convergence of computational methods, etc.

Using this approach, S.I. Lyashko and his colleagues obtained various results regarding optimization problems for different models described by partial differential equations with partial derivatives [1]. In particular, variety of results were obtained for the systems that are described by pseudo-parabolic differential equation.

Let us assume that the evolution of the system is described by the equation $\mathcal{L}u = f$, with a linear integro-differential operator given by

$$\mathcal{L}u \equiv -\sum_{i,j=1}^{n} (a_{ij}(x)\, u_{x_j})_{x_i t} + a(x)u_t - \sum_{i,j=1}^{n} (b_{ij}(x)\, u_{x_j})_{x_i} + b(x)u.$$

Here, the unknown function u describes the system state in the domain $Q = \Omega\times(0,T)$, where $\Omega \subset \mathbb{R}^n$ is a bounded spatial domain with a smooth boundary $\partial\Omega$. The function u satisfies homogeneous Dirichlet-type initial and boundary conditions

$$u|_{t=0} = 0,\ u|_{x\in\partial\Omega} = 0. \tag{1}$$

Let us assume that $\{a_{ij}\}_{i,j=1}^{n}$, $\{b_{ij}\}_{i,j=1}^{n} \subset C^1(\overline{\Omega})$, $a, b \in C(\overline{\Omega})$. In [1] author additionally uses the following assumptions:

$$a_{ij}(x) = a_{ji}(x),\ a(x) \geq 0, \tag{2}$$

$$b_{ij}(x) = b_{ji}(x),\ b(x) \geq 0, \tag{3}$$

there exists $\alpha > 0$, such that functions $a_{ij}(x)$ satisfy inequality

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \alpha \sum_{i=1}^{n} \xi_i^2 \tag{4}$$

for all $\xi_i \in \mathbb{R}$, $i = \overline{1, n}$, and finally

$$\sum_{i,j=1}^{n} b_{ij}(x)\xi_i\xi_j \geq 0 \tag{5}$$

for all $x \in \Omega$.

Based on (2)–(5) author proves a priory inequalities of the form

$$\begin{cases} C_2 \left\| u \right\|_{W_{BR}} \leq \left\| \mathcal{L}u \right\|_{W^-_{BR+}} \leq C_1 \left\| u \right\|_{H_{BR}}, \\ C_2 \left\| v \right\|_{W_{BR+}} \leq \left\| \mathcal{L}^* v \right\|_{W^-_{BR}} \leq C_1 \left\| v \right\|_{H_{BR+}}. \end{cases} \tag{6}$$

Here W_{BR}, H_{BR} denote the completions of the space of smooth functions C^∞, satisfying (1), with respect to the norms

$$\left\| u \right\|_{W_{BR}} = \left(\int_Q u_t^2 + \sum_{i=1}^{n} u_{x_i t}^2 dQ \right)^{\frac{1}{2}},$$

$$\left\| u \right\|_{H_{BR}} = \left(\int_Q u^2 + \sum_{i=1}^{n} u_{x_i}^2 dQ \right)^{\frac{1}{2}},$$

and W^-_{BR}, H^-_{BR} are the corresponding negative spaces with respect to $L_2(Q)$.

In our work, we prove the same inequalities (6) basing only on assumptions (2) and (4), without requiring the fulfillment of conditions (3), (5). Thus, the theorems regarding the correctness of the initial-boundary value problem formulation and the existence of optimal control from [1] remain valid under weaker assumptions.

1. Lyashko S.I. Generalized optimal control of linear systems with distributed parameters. London: Kluwer Academic Publishers, 2002.

ПРО АПРІОРНІ НЕРІВНОСТІ ДЛЯ СИСТЕМ, ЩО ОПИСУЮТЬСЯ ПСЕВДОПАРАБОЛІЧНИМИ ДИФЕРЕНЦІАЛЬНИМИ РІВНЯННЯМИ

У роботі доводяться апріорні нерівності в негативних нормах для систем, що описуються псевдопараболічними диференціальними рівняннями. Базуючись на доведених нерівностях можна стверджувати правильність теорем про коректність постановки початково-крайової задачі, існування оптимального керування, збіжності обчислювальних методів, тощо.