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MODIFICATION OF THE THEORY OF STABILITY OF SOLUTIONS OF DIFFERENTIAL EQUATIONS

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An approach to the modification of the theory of stability of solutions of differential equations and the use of numerical methods for obtaining numerical results in cases where analytical methods become insufficient have been studied. An example of the application of the modified theory and numerical methods for analyzing the stability of solutions of differential equations is given.

The expansion of the theory of stability of solutions of differential equations and the development of numerical methods make it possible to solve more complex problems, such as dynamic systems with complex nonlinear interactions and variable parameters. The application of these modified methods also allows obtaining numerical results with high accuracy and efficiency.

Consider the problem of analyzing the stability of a differential equation with data, which describes the growth of research development over time. Suppose that the equation has the form: $\frac{\partial k}{\partial t} = rk$, where k - population size in time t, and r - is a parameter representing the population growth rate.

Analytically, we can solve this differential equation using separation of variables.[2] The result will look like this: $k(t) = k_0 e^{rt}$, where k_0 - the initial size of the population at the time $t = 0$.

To determine the stability of the solution, you can analyze the sign of the expression e^{rt}.[3] If $r < 0$, then scientific development will decrease to zero at $t \to \infty$, which means stability. If $r > 0$, then scientific development will grow to infinity, which indicates instability.

For numerical analysis, we can use Euler method to approximate the solution. Let's start with the initial value k_0 and we use the Euler method formula: $k_{n+1} = k_n + r\Delta t \cdot k_n$, where Δt is the time step.

For example, if you choose $k_0 = 100$, $r = -0.1$ and $\Delta t = 0.1$, then the value can be calculated k at different time steps and check whether it goes to zero (stability) or to infinity (instability).

So, I considered a differential equation that describes the growth of a population of organisms over time. Analyzing it analytically and numerically, it is possible to obtain information about the sustainability of this development. In the considered problem, I determined that the parameter r, which reflects the growth rate of scientific research, determines its sustainability. This allows us to understand what factors can influence the preservation of scientific development in the future. Therefore, the modification of the theory of stability of solutions of differential equations allows us to expand our capabilities in solving various problems and obtaining a more detailed understanding of the dynamics of systems.

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МОДИФІКАЦІЯ ТЕОРІЇ СТІЙКОСТІ РОЗВ’ЯЗКІВ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ

Досліджено підхід до модифікації теорії стійкості розв'язків диференціальних рівнянь та використання чисельних методів для отримання чисельних результатів у випадках, коли аналітичні методи стають недостатніми. Наведено приклад застосування модифікованої теорії та чисельного методу для аналізу стійкості розв’язків диференціальних рівнянь.