The conference of young scientists «Pidstryhach readings – 2024»
May 27–29, 2024, Lviv

UDC 517.9

EQUILIBRIUM STATES OF DYNAMICAL CONFLICT SYSTEMS WITH A POINT SPECTRUM

Oksana Satur

Institute of Mathematics NAS of Ukraine, orlassat@gmail.com

The main task concerns the question of conditions on initial distributions of dynamical conflict systems that guarantee the emergence of a pure point spectrum in limit distributions (see [1, 2]). According to Theorem 2, the key condition is the choice of at least one opponent's single-priority strategy. This means that the initial distribution of the corresponding opponent should have an advantage over the distribution of the other opponent only in a single local region (see (3)). Another task is to analyze the structure of the point spectrum. A new mathematical model of opinions formation is also constructed using abstract results on the structure of point spectrum. This type of opinions formation model was proposed in the work [3].

The subject of the study is the trajectories of dynamical conflict systems of the following form: $\{\mu^t \equiv \mu_{P^t}, \nu^t \equiv \nu_{R^t}\} \xrightarrow{*} \{\mu^{t+1} \equiv \mu_{P^{t+1}}, \nu^{t+1} \equiv \nu_{R^{t+1}}\}$, $t = 0, 1, \ldots$, where the measures μ^t, ν^t are associated with time-dependent stochastic matrices P^t, R^t which describe the corresponding distributions on Ω for a pair of opponents at each moment of their conflict interaction. Ω is an arbitrary set, which assumes the possibility of applying to it the procedure of infinite shredding, and $\mathcal{B}$ denotes the (Borelian) σ-algebra of all subsets formed by shredding Ω by standard operations, which usually are used in measure theory. In addition to the compactness of $\mathbb{R}^n$, an arbitrary continuous network of neurons, nodes, and abstract hubs is suitable for Ω.

Next, we denote by $\mathcal{M}^{ss}(\Omega)$ the class of measures $\mu = \mu_P$ on $(\Omega, \mathcal{B})$ associative of them with right-infinite stochastic matrices, the columns of which consist of stochastic vectors $\mathbf{p}_k \in \mathbb{R}^n_{1,+}$, $2 \le n < \infty$, $\mathbf{p}_k = (p_{1k}, \ldots, p_{nk})$, $p_{ik} \ge 0$, $i = \overline{1,n}$, $\sum\limits_{i=1}^{n} p_{ik} = 1$. μ-measure on cylinders is determined by the rule $\mu(\Omega_{i_0 \ldots i_k}) := \prod\limits_{l=0}^{k} p_{i_l l}$, and to arbitrary subsets of the σ-algebra $\mathcal{B}$ the measure μ is extended, like the measure λ, in a standard way.

The elements of the first $k \le t$ columns of the matrices P^t, R^t are given by the coordinates $p_{ik} \equiv p_i^k$, $r_{ik} \equiv r_i^k$, $i \in \overline{1,n}$ of vectors $\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^k$, $\mathbf{r}^0, \mathbf{r}^1, \ldots, \mathbf{r}^k$, the coordinates of which are determined according to the formulas:

$$p_i^k = p_i^{k-1} \cdot \frac{1 - r_i^{k-1}}{1 - \theta^{k-1}}, \quad r_i^k = r_i^{k-1} \cdot \frac{1 - p_i^{k-1}}{1 - \theta^{k-1}}. \tag{1}$$

Next, we will assume that in the general case the coordinates $p_i^0 \equiv p_{0i}$, $r_i^0 \equiv r_{0i}$ are strictly positive and different:

$$p_i^0,\ r_i^0 > 0, \quad p_i^0 \neq r_i^0, \quad \theta^0 = (\mathbf{p}^0, \mathbf{r}^0) \neq 1. \tag{2}$$

Theorem 1. *Let $\mathbf{p}^0, \mathbf{r}^0 \in \mathbb{R}^{n=2}_{+,1}$. Then, under the condition* (2) *both measures μ^∞, ν^∞ associated to the limit matrices P^∞, R^∞ are purely point, $\mu^\infty, \nu^\infty \in \mathcal{M}_{\mathrm{pp}}$.*

Theorem 2. *Let $\mathbf{p}^0, \mathbf{r}^0 \in \mathbb{R}^{n>2}_{+,1}$. Then, under the condition* (2) *one of the limit measures μ^∞, ν^∞ will be purely point, $\mu^\infty \in \mathcal{M}_{\mathrm{pp}}$, або $\nu^\infty \in \mathcal{M}_{\mathrm{pp}}$, if there exists only one index $1 \leq \mathbf{i} \leq n$, for vector $\mathbf{p}^0$, or $1 \leq \mathbf{j} \leq n$, for vector $\mathbf{r}^0$,such that one of the inequalities holds, respectively:*

$$p^0_{\mathbf{i}} > r^0_{\mathbf{i}}, \qquad p^0_{\mathbf{j}} < r^0_{\mathbf{j}}. \tag{3}$$

At the same time, if $\mu^\infty \in \mathcal{M}_{\mathrm{pp}}$, then $\nu^\infty \in \mathcal{M}_{\mathrm{sc}}$ and vice versa if $\nu^\infty \in \mathcal{M}_{\mathrm{pp}}$, then $\mu^\infty \in \mathcal{M}_{\mathrm{sc}}$. If none of the inequalities (3) *holds, then both limit measures are singularly continuous: μ^∞, $\nu^\infty \in \mathcal{M}_{\mathrm{sc}}$. In any case, the limit measures μ^∞, ν^∞ invariant with respect to the transformation $\circledast$.*

The author expresses gratitude for partial financial support for the project "Mathematical modeling of complex dynamical systems and processes relevant to state security"(No. 0123U100853).

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РІВНОВАЖНІ СТАНИ ДИНАМІЧНИХ КОНФЛІКТНИХ СИСТЕМ З ТОЧКОВИМ СПЕКТРОМ

Досліджується структура точкового спектру в граничних по часу станах динамічних систем конфлікту в термінах ймовірнісних мір. Показано, що необхідною і достатньою умовою виникнення мір з точковим спектром є стратегія єдиного пріоритету. В цьому випадку встановлена експоненційна швидкість концентрації розподілів з точковим спектром та його щільність у фазовому просторі. Запропонована можливість застосування інформації про структуру точкового спектру в новій математичній моделі формування переконань у індивідів абстрактного суспільства.