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LINEAR CONJUGATION PROBLEM WITH NONLOCAL CONDITIONS FOR MIXED FACTORIZED HIGHER ORDER EQUATIONS

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Let $\Omega = \mathbb{R}/2\pi\mathbb{Z}$ be a unit circle and let $\mathcal{D} = (-\alpha, \beta) \times \Omega$ be a cylindrical domain of the variables (t, x), that is separated by the hyperplane $\{t = 0\} \times \Omega$ into nonoverlapping cylindrical subdomains $\mathcal{D}_- = (-\alpha, 0) \times \Omega$ and $\mathcal{D}_+ = (0, \beta) \times \Omega$, where α and β are positive real numbers.

The problem we aim to solve is finding a pair of functions $u_1 = u(t, x)$ and $u_2 = u_2(t, x)$, defined in $\mathcal{D}_-$ and $\mathcal{D}_+$, respectively, which satisfy the following differential equations

$$\begin{cases} \prod\limits_{j=1}^{n} \Big(\dfrac{\partial}{\partial t} - \lambda_j \dfrac{\partial}{\partial x}\Big) u_1 = 0, \quad (t, x) \in \mathcal{D}_-, \\ \prod\limits_{j=1}^{m} \Big(\dfrac{\partial}{\partial t} - \mu_j \dfrac{\partial}{\partial x}\Big) u_2 = 0, \quad (t, x) \in \mathcal{D}_+, \end{cases} \tag{1}$$

with conjugate conditions

$$\lim_{t\to 0_-} \frac{\partial^{j-1} u_1}{\partial t^{j-1}} = \gamma_j \lim_{t\to 0_+} \frac{\partial^{j-1} u_2}{\partial t^{j-1}}, \quad j = 1, \ldots, m, \quad x \in \Omega, \tag{2}$$

nonlocal conditions

$$\frac{\partial^{j-1} u_1}{\partial t^{j-1}}\bigg|_{t=-\alpha} - \nu_j \, \frac{\partial^{j-1} u_2}{\partial t^{j-1}}\bigg|_{t=\beta} = \varphi_j(x), \quad j = 1, \ldots, m, \quad x \in \Omega, \tag{3}$$

and initial conditions

$$\frac{\partial^{m+j-1} u_1}{\partial t^{m+j-1}}\bigg|_{t=-\alpha} = \varphi_{m+j}(x), \quad j = 1, \ldots, n-m, \quad x \in \Omega, \tag{4}$$

where $n, m \in \mathbb{N}$, $1 \le m \le n$, $\lambda_j, \mu_j \in \mathbb{R}\backslash\{0\}$, $\gamma_j, \nu_j \in \mathbb{C}$, $\varphi_j(x)$ are given functions. Moreover, we suppose that numbers $\lambda_1, \ldots, \lambda_n$ as well as $\mu_1, \ldots, \mu_m$ are pairwise different, respectively.

In general, this problem are conditionally well-posed and its solvability is related with the problem of small denominators and may be unstable with respect to small variations in the coefficients of the problem and in the parameters of the domain. Using the Fourier method of separation of variable and metric approach [1, 2], we will be discuss the conditions for the solvability of the problem (1)–(4) in Sobolev spaces and the proving estimates for small denominators for almost all (with respect to the Lebesgue measure in space $\mathbb{R}^m$) vectors $(\mu_1, \ldots, \mu_m)$ or almost all (with respect to the Lebesgue measure in space $\mathbb{R}^n$) vectors $(\lambda_1, \ldots, \lambda_n)$.

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ЗАДАЧА ЛІНІЙНОГО СПРЯЖЕННЯ З НЕЛОКАЛЬНИМИ УМОВАМИ ДЛЯ МІШАНИХ ФАКТОРИЗОВАНИХ РІВНЯНЬ ВИСОКОГО ПОРЯДКУ

Із використанням метричного підходу досліджуються умови єдиності та існування розв'язку у просторах Соболева задачі лінійного спряження з нелокальними умовами для мішаних факторизованих рівнянь високого порядку у циліндричній області.