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ABSTRACTS

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Abstracts of XI International V.Skorobohatko Mathematical Conference are published. The new results in a few branches of mathematics relevant to interests of Prof. Vitaliy Skorobohatko (1927-1996) are presented. Tasks in the fields of ordinary differential equations and differential equations with partial derivatives are considered, problems in function theory, functional analysis, algebra and computational mathematics are analyzed. A number of applications to problems in mathematical physics and mechanics are developed.

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First page: portrait of V.Skorobohatko

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ON FINITE SEMI-DISTRIBUTIVE NEARRINGS

Nearrings are generalized associative rings in which the addition need not be commutative and only one (left or right) distributive law is assumed. A left nearring R with identity is said to be *semi-distributive* if $(r+s+r)t = rt+st+rt$ for all $r, s, t \in R$. Clearly every associative ring is a semi-distributive nearring, but not conversely.

Proposition. *Every nearring with identity whose additive group is an elementary abelian* 2*-group is semi-distributive.*

As an example, there exist 4274 non-isomorphic nearrings with identity of order 16, among which 3080 are semi-distributive. The next table is obtained by the package SONATA of the computer algebra system GAP.

Additive groups	Number of semi-distributive nearrings
C_{16}	1
$C_4 \oplus C_4$	10
$C_8 \oplus C_2$	7
$C_4 \oplus C_2 \oplus C_2$	264
$C_2 \oplus C_2 \oplus C_2 \oplus C_2$	2798

It was proved by the first author that every semi-distributive nearring of odd order is a ring. Therefore, the study of such nearrings can be reduced to the case of additive 2-groups. The following theorem is a step in this direction. We recall that a nearring is simple if it has no proper ideals. As well-known, every finite simple ring is isomorphic to a complete matrix ring over a finite field.

Theorem. *A finite semi-distributive nearring R is simple if and only if R is a finite simple associative ring.*

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ON ALMOST PSEUDO-CONFORMALLY SYMMETRIC RIEMANNIAN MANIFOLDS

The object of the present paper is to study a type of non-flat Riemannian manifolds called almost pseudo-conformally symmetric Riemannian manifolds and find out some fruitful and interesting results on it. The existence of an almost pseudo-conformally symmetric Riemannian manifold is also shown by a non-trivial example.

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OPTIMIZATION OF MULTI-ELEMENT PHASE FIELD TRANSFORMERS

The problem of optimization of the radiation characteristics of the multi-element field transformer consists of solving multiparameter nonlinear integral or matrix equations. The problem is redused do maximization of functional that has clear physical meaning in the form

$$\chi(R_1, R_2..., R_M; T_1, T_2..., T_{M-1}) = \int_{-\theta_0}^{\theta_0} |f(\theta)|\, F(\theta) d\theta. \qquad (1)$$

Function $f(\theta)$ is radiation pattern of whole system and it is presented as sum of partial pattern of its elements

$$f(\theta) = \sum_{m=1}^{M} f_m(\theta), \qquad m = 1, ..., M, \qquad (2)$$

where the angle θ is measured counter-clockwise from the geometrical direction of the reflected rays making up the angle 2β with the inverse z-axis. The partial patterns $f_m(\theta)$ are defined by the fields on the mirrors and their reflection factors.

In the physical sense, the problem consists of creating the desired amplitude radiation pattern $F(\theta)$, $\theta \in [-\theta_0, \theta_0]$, at the given amplitude $U_0(x_0)$, $x_0 \in [-1, 1]$ of the input field by choosing appropriate complex reflection coefficients $R_m(x_m)$ and transmission ones $T_m(x_m)$. It is assumed that these coefficients fulfill the conditions

$$|R_m(x_m)|^2 + |T_m(x_m)|^2 = 1, \qquad m = 1, ..., M. \qquad (3)$$

Maximization of (1) is carried out by choosing the parameters T_m and R_m in such a way that its value increases in the each iteration.

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ON ELLIPTIC PROBLEMS WITH BOUNDARY DATA IN GENERALIZED SOBOLEV SPACES

The talk is devoted to applications of the Hilbert generalized Sobolev spaces H^{α}, where $\alpha \in \mathrm{OR}$, to elliptic problems whose boundary data are arbitrary distributions. Here, OR is the set of all Borel measurable functions $\alpha : [1,\infty) \to (0,\infty)$ such that $c^{-1} \leq \alpha(\lambda t)/\alpha(t) \leq c$ whenever $t \geq 1$ and $\lambda \in [1,2]$, with $c \geq 1$ not depending on t and λ. These functions are called O-regularly varying at infinity. The space $H^{\alpha}(\mathbb{R}^n)$ consists of all tempered distributions $w \in \mathcal{S}'(\mathbb{R}^n)$ whose Fourier transform Fw satisfies $\alpha(1+|\xi|) \cdot (Fw)(\xi) \in L_2(\mathbb{R}^n, d\xi)$. Its analogs for Euclidean domains and smooth compact manifolds are defined standardly.

Given a bounded domain $\Omega \subset \mathbb{R}^n$ with $\Gamma := \partial\Omega \in C^{\infty}$, we consider a regular differential elliptic problem $Au = f$ in Ω and $B_j u = g_j$ on Γ with $j = 1, ..., q$. Here, $\mathrm{ord}\, A = 2q$, and $m_j := \mathrm{ord}\, B_j$, and all coefficients of A and B_j belong to $C^{\infty}(\overline{\Omega}, \mathbb{C})$ or $C^{\infty}(\Gamma, \mathbb{C})$, resp. The boundary data are supposed to be arbitrary distributions (generally, of low regularity). Thus, each $g_j \in H^{\varphi_j}(\Gamma)$ for some $\varphi \in \mathrm{RO}$, where $\varphi_j(t) \equiv \varphi(t) t^{2q-m_j-1/2}$.

We indicate parameters $\eta \in \mathrm{RO}$ such that the elliptic problem is Fredholm on the pair of Hilbert spaces $\{u \in H^{\varphi\varrho^{2q}}(\Omega) : Au \in H^{\eta}(\Omega)\}$ and $H^{\eta}(\Omega) \oplus H^{\varphi_1}(\Gamma) \oplus \cdots \oplus H^{\varphi_q}(\Gamma)$, with $(\varphi\varrho^{2q})(t) \equiv \varphi(t) t^{2q}$ and the source space being induced with the graph norm. We also prove a theorem on isomorphisms induced by the elliptic problem between subspaces of these spaces and theorems on local (up to Γ) regularity of generalized solutions to the problem and their local *a priory* estimates in the spaces used. We discuss applications of these theorems to homogeneous elliptic equations, quadratic interpolation between Hilbert function spaces associated with these equations, and to elliptic problems with white boundary noise.

These results are obtained together with R. Denk [1].

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TRUNCATION ERROR BOUNDS FOR BRANCHED CONTINUED FRACTION OF SPECIAL FORM

Let N be a fixed integer number, $i(k)=(i_1,i_2,\ldots,i_k)$ be a multiindex and let $\mathcal{I}=\{i(k):\ 1\le i_p\le i_{p-1},\ 1\le p\le k,\ i_0=N,\ k\in\mathbb{N}\}$ be a set of multiindices.

We study the convergence of a branched continued fraction (BCF)

$$\sum_{i_1=1}^{N}\frac{a_{i(1)}}{1}{+}\sum_{i_2=1}^{i_1}\frac{a_{i(2)}}{1}{+}\sum_{i_3=1}^{i_2}\frac{a_{i(3)}}{1}{+}\cdots, \tag{1}$$

where $a_{i(k)}$, $i(k)\in\mathcal{I}$, are complex numbers.

Let the elements of the BCF (1) satisfy the following inequalities

$$-\mu_{i_k}\cos\alpha_{i_{k-1}}\le(i_{k-1}+1)\mathrm{Re}(a_{i(k)}e^{i(\alpha_{i_{k-1}}+\alpha_{i_k})})\le\mu_{i_k}|\sin\alpha_{i_{k-1}}|,$$

$$\mathrm{Im}(a_{i(k)}e^{i(\alpha_{i_{k-1}}+\alpha_{i_k})})\ge0,\quad |a_{i(k)}|\le L$$

for all $i(k)\in\mathcal{I}$ and $k\ge2$, where L is positive constant,

$$\mu_k=\max\left\{\left(\cos^2\alpha_k+\frac{1}{(k+1)^2}\right)^{1/2}|\sin\alpha_k|,\ \frac{1}{k+1},\ \frac{k+2}{2k+2}|\sin2\alpha_k|\right\},$$

$\alpha_k\in(0;\pi/2)$, $1\le k\le N$. Then the BCF (1) converges to a finite value f and the inequalities

$$|f-f_n|\le C_{N+n}^{N-1}\frac{L^n}{d_0(L^2+d)^{n/2}}\max_{i(1)\in\mathcal{I}}|a_{i(1)}|,\quad n\ge1, \tag{2}$$

where $d_0=\min\limits_{1\le k\le N}\mu_k$ and $d=\dfrac{1}{4}\min\limits_{i(2)\in\mathcal{I}}(\mu_{i_2}\sin2\alpha_{i_1})^2$, are valid.

If conditions $\mathrm{Im}(a_{i(k)}e^{i(\alpha_{i_{k-1}}+\alpha_{i_k})})\ge0,\ i(k)\in\mathcal{I},\ k\ge2$, and $\alpha_k\in(0;\pi/2)$, $1\le k\le N$, are replaced by conditions

$$\mathrm{Im}(a_{i(k)}e^{i(\alpha_{i_{k-1}}+\alpha_{i_k})})\le0,\ i(k)\in\mathcal{I},\ k\ge2;\ \alpha_k\in(-\pi/2;0),\ 1\le k\le N,$$

then the inequalities (2) are also valid.

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A TWO-POINT BOUNDARY VALUE PROBLEM FOR SECOND ORDER DIFFERENTIAL EQUATION WITH PIECEWISE-CONSTANT ARGUMENT OF GENERALIZED TYPE

On $[0, T]$ consider two-point boundary value problem for second order differential equation with piecewise-constant argument of generalized type

$$\ddot{x} = a_1(t)\dot{x}(t) + a_2(t)x(t) + a_3(t)\dot{x}(\gamma(t)) + a_4(t)x(\gamma(t)) + f(t), \qquad (1)$$

$$b_{11}\dot{x}(0) + b_{21}x(0) + c_{11}\dot{x}(T) + c_{21}x(T) = d_1, \qquad (2)$$

$$b_{12}\dot{x}(0) + b_{22}x(0) + c_{12}\dot{x}(T) + c_{22}x(T) = d_2, \qquad (3)$$

where $x(t)$ is unknown function, the functions $a_i(t)$, $i = \overline{1, 4}$ and $f(t)$ are continuous on $[0, T]$; $0 = \theta_0 < \theta_1 < ... < \theta_{N-1} < \theta_N = T$, $\theta_j \leq \zeta_j \leq \theta_{j+1}$ for all $j = 0, 1, ..., N-1$: $\gamma(t) = \zeta_j$ if $t \in [\theta_j, \theta_{j+1})$, $j = \overline{0, N-1}$; b_{sp}, c_{sp} and d_s are constants, where $s, p = 1, 2$.

A solution to problem (1)-(3) is a function $x(t)$ is twice continuously differentiable on $[0, T]$, it satisfies the equation (1) and boundary conditions (2), (3).

Differential equations with piecewise-constant argument of generalized type are introduced in [1]. Examples of the applications of these equations to the various problems have been under intensive investigation for the last decades. We study conditions for well-posedness of problem (1)-(3) and construct the algorithms for finding its solution. For this we use the Dzhumabaev parameterization method [2].

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ON GENERIC INHOMOGENEOUS BOUNDARY-VALUE PROBLEMS IN SOBOLEV SPACES

We consider the most general class of Fredholm one-dimensional inhomogeneous boundary-value problems for systems of linear ordinary differential equations of an arbitrary order whose solutions and right-hand sides belong to appropriate Sobolev spaces. Boundary conditions of these problems may contain derivatives the order of higher than the order of the system of differential equations. It is established that each of these boundary-value problems correspond to a certain rectangular numerical characteristic matrix with kernel and cokernel having the same dimension as the kernel and cokernel of the boundary-value problem. The assumption under which the sequence of characteristic matrices to converge are found. For parameter-dependent problems from this class, we prove a constructive criterion for their solutions to be continuous in the Sobolev space with respect to the parameter. We also prove a two-sided estimate for the degree of convergence of these solutions to the solution of the nonperturbed problem.

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ABOUT TRAVELLING WAVES IN FERMI-PASTA-ULAM SYSTEM ON 2D-LATTICE

We study the Fermi–Pasta–Ulam system that describes the dynamics of an infinite system of nonlinearly coupled particles on two dimensional lattice

$$\begin{gathered}\ddot{q}_{n,m}(t) = W_1'(q_{n+1,m}(t) - q_{n,m}(t)) - W_1'(q_{n,m}(t) - q_{n-1,m}(t)) + \\ + W_2'(q_{n,m+1}(t) - q_{n,m}(t)) - W_2'(q_{n,m}(t) - q_{n,m-1}(t)),\ (n,m) \in \mathbb{Z}^2,\end{gathered} \tag{1}$$

where $q_{n,m} = q_{n,m}(t)$ is a coordinate of (n,m)-th particle at time t, $W_1, W_2 \in C^1(\mathbb{R})$ are the potentials of neighbor interactions.

Travelling wave is a solution of the form

$$q_{n,m}(t) = u(n\cos\varphi + m\sin\varphi - ct),$$

where the function $u(s)$, $s = n\cos\varphi + m\sin\varphi - ct$, is called the profile function, or simply profile, of the wave and the constant $c \neq 0$ the speed of the wave. Making use the travelling wave we obtain the equation

$$\begin{gathered}c^2u''(s) = W_1'(u(s+\cos\varphi) - u(s)) - W_1'(u(s) - u(s-\cos\varphi)) + \\ + W_2'(u(s+\sin\varphi) - u(s)) - W_2'(u(s) - u(s-\sin\varphi)),\end{gathered} \tag{2}$$

for the profile function $u(s)$. This equation has, actually, a variational structure.

We obtain, by means of the critical points method, a results on the existence of nonconstant periodic and solitary travelling waves.

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ON EXISTENCE OF MAIN POLYNOMIAL FOR ANALYTIC VECTOR-VALUED FUNCTIONS OF BOUNDED **L**-INDEX IN THE UNIT BALL

For $R = (r_1, r_2) \in \mathbb{R}^2_+ := (0, +\infty)^2)$ we denote by $\mathbb{D}^2((z_0, \omega_0), R) = \{(z, \omega) \in \mathbb{C}^2 : |z - z_0| < r_1, |\omega - \omega_0| < r_2\}$ a polydics, and $\mathbb{B}^2((z_0, \omega_0), r) = \{(z, \omega) \in \mathbb{C}^2 : \sqrt{|z - z_0|^2 + |\omega - \omega_0|^2} < r\}$ a ball of the radii $r > 0$, $\mathbb{B}^2 = \mathbb{B}^2((0, 0), 1)$. An analytic vector-valued function $F = (f_1, f_2) : \mathbb{B}^2 \to \mathbb{C}^2$ is said to be *of bounded* **L***-index in joint variables*, if there exists $n_0 \in \mathbb{Z}_+$ such that $(\forall (z, \omega) \in \mathbb{B}^2)$ $(\forall (i, j) \in \mathbb{Z}^2_+)$

$$\frac{\|F^{(i,j)}(z,\omega)\|}{i!j!l_1^i(z,\omega)l_2^j(z,\omega)} \leq \max\left\{\frac{\|F^{(k,m)}(z,\omega)\|}{k!m!l_1^k(z,\omega)l_2^m(z,\omega)} : k, m \in \mathbb{Z}_+, k + m \leq n_0\right\}.$$

Here $\|F\| = \max\{|f_j| : j \in \{1, 2\}\}$. et $(z_0, w_0) \in \mathbb{B}^2$. We expand a vector-function $F : \mathbb{B}^2 \longrightarrow \mathbb{C}^2$ in vector-valued power series

$$F(z, w) = \sum_{k=0}^{\infty} P_k(z - z_0, w - w_0) = \sum_{k=0}^{\infty} \sum_{i+j=k} B_{ij}(z - z_0)^i (w - w_0)^j, \quad (1)$$

where P_k is a homogeneous polynomial of degree k.

Theorem. *Let* $\mathbf{L} \in Q(\mathbb{B}^2)$*. If an analytic vector-function* $F : \mathbb{B}^2 \to \mathbb{C}^2$ *has bounded* L*-index in joint variables then there exists* $p \in \mathbb{Z}_+$ *such that for each* $d \in \left(0; \frac{\beta}{\sqrt{2}}\right]$ *there exists* $\eta(d) \in (0; d)$ *such that for each* $(z_0, w_0) \in \mathbb{B}^2$ *and for some* $r = r(d, (z_0, w_0)) \in (\eta(d), d)$ *and some* $\nu_0 = \nu_0(d, (z_0, w_0)) \leq p$ *the polynomial* p_{ν_0} *is main in series* (1) *on* $\mathbb{T}^2\left((z_0, w_0), \frac{r\mathbf{1}}{\mathbf{L}(z_0, w_0)}\right)$.

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THE NONLOCAL MULTIPOINT PROBLEM WITH DIRICHLET-TYPE CONDITIONS FOR AN ORDINARY DIFFERENTIAL EQUATION OF EVEN ORDER WITH INVOLUTION

The spectral properties of the nonself-adjoint problem with multipoint perturbations of the Dirichlet conditions for differential operator of order $2n$ with involution are investigated. The system of eigenfunctions of a multipoint problem is constructed. Sufficient conditions have been established, under which this system is complete and, under some additional assumptions, forms the Riesz basis.

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MATHEMATICAL MODELING OF THE STOKES PARAMETERS IN SN 1006

Supernova remnants are believed to be the best candidates for sources of cosmic rays up to energy $\sim 10^{15}$ eV. They are observed in all electromagnetic domains from radio to very-high-energy gamma-rays. At present, mostly the observed spectra or surface brightness distribution are used for the analysis of accelerated particles on shock waves of supernova remnants. Other important observational data, namely, the radio polarization maps are almost out of use.

We have developed theoretical model and performed numerical three-dimensional magneto-hydrodynamic simulations in order to simulate the radio polarization maps of SN 1006. The Faraday rotation and ambient medium with non-uniform distribution of magnetic field are taken into account in our model. We have also included the randomization of the disordered magnetic field in SN1006.

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ON THE CONVERGENCE OF MULTIDIMENSIONAL S-FRACTIONS WITH INDEPENDENT VARIABLES

Let

$$1+\mathop{\mathrm{D}}_{k=1}^{\infty}\sum_{i_k=1}^{i_{k-1}}\frac{c_{i(k)}z_{i_k}}{1},\tag{1}$$

where $z_{i_k}\in\mathbb{C},\ i_k=1,2,...,N;\ \ c_{i(k)}>0,\ i(k)\in\mathcal{I},$

$$\mathcal{I}=\{i(k)=(i_1,i_2,\ldots,i_k)\ :1\le i_k\le i_{k-1}\le ...\le i_0;\ k\ge 1;\ i_0=N\},$$

be a multidimensional S-fraction with independent variables, N is a fixed natural number.

Theorem. *Let the elements of a BCF (1) satisfy the conditions*

$$c_{i(k)}\le c,\ i(k)\in\mathcal{I}.$$

Then the multidimensional S-fraction with independent variables (1) converges uniformly on every compact subset K of

$$D=\left\{\mathbf{z}\in\mathbb{C}^N\setminus\{0\}:-\frac{\pi}{2}<\arg z_N\le\arg z_{N-1}\le\ldots\le\arg z_1<0\right\}$$

to a holomorphic function in D and the following truncation error bound holds

$$|f_m(\mathbf{z})-f_{Nn}(\mathbf{z})|<D_N\left(\frac{\sqrt{\delta^2+4M}-\delta}{\sqrt{\delta^2+4M}+\delta}\right)^n,\ m\ge Nn,\ n\ge 1,$$

where $M=c\max\limits_{\mathbf{z}\in K,1\le m\le N}|z_m|,\ \delta=\cos\left(\max\limits_{\mathbf{z}\in K}|\arg z_N|\right),$

$$D_1=2M/\delta,\ D_r=4\left(D_{r-1}S+M/\delta\right),\ r=2,\ldots,N,$$

$$S=1+M\sqrt{M^2+\delta^4}/\left(\delta^2\left(\sqrt{M^2+\delta^4}-M\right)\right).$$

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STABLE RANGE OF SOME CLASSES OF NON COMMUTATIVE RINGS

It is proved that the right adequate duo ring with a nonzero Jacobson radical has a stable range one. It is established that the class of full matrices of order 2 over Hermite duo ring stable range one.

Theorem 1. *Let R be the right adequate duo ring with a nonzero Jacobson radical. Then stable range of a ring R is equal* 1.

Theorem 2. *Let R be the right adequate duo ring. Then stable range of a ring R is equal* 2.

Theorem 3. *Let R be Hermite duo ring . If $aR + xR = R$ and $cR + zR + (ay - bx)R = R$*, then for matrices

$$A = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix}, B = \begin{pmatrix} x & 0 \\ y & z \end{pmatrix}$$

there exists such invertible matrices U, V, that $AU + BV = E$, where E is an identity matrix.

Denote by $M_2(R)$ the ring of matrices of the second order over the duo ring R. We will say that the matrix $A \in M_2(R)$ is full if $M_2(R)AM_2(R) = M_2(R)$. The set of all full matrices of the ring $M_2(R)$ is denoted by $F_2(R)$.

Theorem 4. *Let R be Hermite duo ring and $A, B \in M_2(R)$ such, that $AM_2(R) + BM_2(R) = M_2(R)$. Then there exist such full matrix $T \in F_2(R)$, that $A + BT$ is invertible matrix.*

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BRANCHED CONTINUED FRACTIONS WITH INDEPENDENT VARIABLES

Branched continued fractions (BCF) is a multidimensional generalization of continued fractions. They were firstly defined by V. Ya. Skorobohatko in 1967. The strict definition of those fractions based on compositions of linear fractional transformation was given by P. I. Bodnarchuk. In present, three classes of BCF have been distinguished: general branched continued fractions with N branches of branching, two-dimensional continued fractions, branched continued fractions with independent variables.

Two-dimensional continued fractions appeared as a result of consideration of the correspondence between BCF and double power series. This type of fraction was an object of research of Kh. Yo. Kuchminska, O. M. Sus, T. M. Antonova, S. M. Vozna, M. M. Pahirya, J. A. Murphy, M. R. ODonohoe, W. Siemaszko, A. Cuyt. In the case of N variables, the construction of that fractions is quite bulky.

For a long time the question of convergence of general BCF had been opened, in particular, in present, the multidimensional generalization of Seidels convergence criterion for continued fractions with real positive elements is unknown. To solve that issue there was proposed to simplify the structure of BCF. Therefore, it was done by D. I. Bodnar in 1976 in the case of N=2. Later, BCF of that structure were called branched continued fractions with independent variables, and if variables in those fractions are fixed then they are called BCF of the special form. In the general case, a multidimensional generalization of Seidels criterion for BCF of the special form was established by I. B. Bilanyk in 2017. Works of D. I. Bodnar, R. I. Dmytryshyn, Kh. Yo. Kuchminska, O. E. Baran, T. M. Antonova, M. M. Bubniak, I. B. Bilanyk, W. Siemaszko are dedicated to developing the analytic theory of these fractions. BCF with independent variables have appeared to be effective, in particular, for solving the problem of correspondence between multiple power series and branched continued fractions.

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INITIAL-BOUNDARY VALUE PROBLEMS FOR NONLINEAR PARABOLIC EQUATIONS IN UNBOUNDED DOMAINS

Let Ω be a unbounded domain in $\mathbb{R}^n\,(n\in\mathbb{N})$ with the piecewise smooth boundary $\partial\Omega$, $\partial\Omega=\Gamma_0\cup\Gamma_1$, $\Gamma_0\cap\Gamma_1=\emptyset$, $\nu=(\nu_1,\ldots,\nu_n)$ be a unit outward pointing normal vector on the $\partial\Omega$. Put $Q:=\Omega\times(0,T)$, $\Sigma_0:=\Gamma_0\times(0,T)$, $\Sigma_1:=\Gamma_1\times(0,T)$, where $T>0$.

A *partial case* of considered problems is to find a function $u:\overline{Q}\to\mathbb{R}$ such that

$$u_t-\sum_{i=1}^{n}\big(a_i(x,t)|u_{x_i}|^{p_i(x)-2}u_{x_i}\big)_{x_i}+a_0(x,t)|u|^{p_0(x)-2}u=f(x,t),\ (x,t)\in Q,$$

$$u\Big|_{\Sigma_0}=0,\quad \sum_{i=1}^{n}|u_{x_i}|^{p_i(x)-2}u_{x_i}\nu_i\Big|_{\Sigma_1}=0,$$

$$u(x,0)=u_0(x),\quad x\in\Omega.$$

Here $p_i\in L_\infty(\Omega)$, $\operatorname{ess\,inf}_{x\in\Omega}p_i(x)>1$, $i=\overline{0,n}$, $a_i\in L_\infty(Q)$, $i=\overline{0,n}$, are positive functions, and $u_0:\Omega\to\mathbb{R}$, $f:Q\to\mathbb{R}$ are given real-valued integrable functions.

As is well known, in order to guarantee the uniqueness of the solutions of the initial-boundary value problems for the linear and the some nonlinear parabolic equations in unbounded domains, we need to impose certain restrictions on their growth to infinity at $|x|\to+\infty$, for example, to require limitations of solutions or their belonging to some functional spaces. However, there are nonlinear parabolic equations, for which initial-boundary value problem is uniquely solvable without any conditions at infinity.

This talk is devoted the *existence and uniqueness of weak solutions* of considered problems (from the generalized Lebesgue and Sobolev spaces) either with conditions on behavior of solutions and growth of data-in at infinity or without it.

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ON DEFORMATION CENTERS OF DYNKIN DIAGRAMS

Through, all graphs $G = (G_0, G_1)$ are connected, undirected, without cycles and with $1 < |G_0| < \infty$. $\mathbb{R}^+ = \{x \in \mathbb{R} \,|\, x > 0\}$.

By an edges-weighted graph $G = (G_0, G_1, \varphi)$ we mean a graph (G_0, G_1) together with a function $\varphi : G_1 \to \mathbb{R}^+$. The φ-distance $d_\varphi(u, v)$ between vertices u and $v \neq u$ is $\sum_{i=1}^{s} \varphi(\lambda_i)$ for the path $\lambda = \lambda_1 \lambda_2 \cdots \lambda_s$ between them (λ_i are edges). The eccentricity of a vertex is the maximum φ-distance from it to any other vertex. The maximum (resp. minimum) eccentricity among all vertices is called the diameter (resp. radius) of G. The center of G is the set of all vertices of minimum eccentricity (called central). $G = (G_0, G_1, \varphi)$ is said to be Aut-central if there is the only central vertex up to automorphism of G.

Let now $G = (G_0, G_1)$ be a Dynkin diagram and $s \in G_0$. The s-th P-limiting number $\mathrm{pld}(s)$ of G is the greatest number $a \in \mathbb{R}$ for which the quadratic form

$$q_G^{(s)}(z, a) = q_Q(z_1, z_2, \ldots, z_n, a) := a z_s^2 + \sum_{i \in G_0 \setminus s} z_i^2, - \sum_{\{i-j\} \in G_1} z_i z_j$$

called the s-th deformation of the (positive) quadratic Tits form $q_G(z) \simeq q_G^{(s)}(z, 1)$ of G, is not positive ($\mathrm{pld}(s) \geq 0$, $\mathrm{pld}(s) = 0$ iff $|G_0| = 1$) [1].

We study Dynkin diagrams $G = (G_0, G_1)$ as the edges-weighted graphs $G = (G_0, G_1, \mathrm{pld}^+)$, where, for any edge σ between vertices u and v, $\mathrm{pld}^+(\sigma) = \mathrm{pld}(u) + \mathrm{pld}(v)$.

In particular we prove the following theorem.

Theorem. *Any edges-weighted Dynkin diagram $G = (G_0, G_1, \mathrm{pld}^+)$ is Aut-central.*

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HYPERBOLIC STOKES SYSTEM WITH VARIABLE EXPONENT OF NONLINEARITY

Let $n \in \mathbb{N}$ and $T > 0$ be fixed numbers, $n \geq 2$, $\Omega \subset \mathbb{R}^n$ be a bounded domain with the smooth boundary $\partial\Omega$, $Q_{0,T} = \Omega \times (0, T)$.

We consider the problem of finding a pair of functions $\{u, \pi\}$ that satisfy the following relations:

$$u_{tt} - \sum_{i,j=1}^{n} \Big(A_{ij}(x,t)u_{x_i}\Big)_{x_j} + G(x,t)|u_t|^{q(x,t)-2}u_t +$$

$$+\nabla\pi = f(x,t) \quad \text{in} \quad Q_{0,T}, \tag{1}$$

$$\operatorname{div} u = 0 \quad \text{in} \quad Q_{0,T}, \tag{2}$$

$$\int_{\Omega} \pi(x,t)\, dx = 0 \quad \text{in} \quad (0,T), \tag{3}$$

$$u|_{\partial\Omega\times[0,T]} = 0, \tag{4}$$

$$u|_{t=0} = u_0(x) \quad \text{in} \quad \Omega, \tag{5}$$

$$u_t|_{t=0} = u_1(x) \quad \text{in} \quad \Omega, \tag{6}$$

where $u = (u_1, \ldots, u_n) : Q_{0,T} \to \mathbb{R}^n$ is the velocity field, $\pi : Q_{0,T} \to \mathbb{R}$ is the pressure, $\nabla\pi = (\frac{\partial\pi}{\partial x_1}, ..., \frac{\partial\pi}{\partial x_n})$, $\operatorname{div} u = \frac{\partial u_1}{\partial x_1} + ... + \frac{\partial u_n}{\partial x_n}$, A_{ij}, G are some matrix, f is some vector function. The function $q(\geq 1)$ is called the variable exponent of nonlinearity to the equation (1).

Under additional conditions for data-in, we prove the solvability of problem (1)–(6).

The corresponding parabolic Stokes system with variable exponent of nonlinearity was studied by O. M. Buhrii in [1].

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A DARK ENERGY MODEL INSPIRED BY FIELD THEORY AND ITS COSMOLOGICAL CONSEQUENCES

In 1998, the cosmology experienced a breakthrough after discovery of the accelerated expansion of the universe through the analysis of distant Type Ia supernovae data and afterwards, an exotic matter "dark energy" (DE) (reviewed in [1]), characterized by negative pressure, has been proposed as the driving force behind the accelerated expansion. Although Λ is the simplest candidate of DE, other phenomenological models have also been considered by the theorists to describe the cosmology of the late time acceleration of the universe. Scalar field models are the phenomenological models motivated by field theory, where during the phase of not rolling down they act as cosmological constant. Tachyon, quintessence, phantom and quintom are such scalar field models.

The reconstruction scheme is as follows. We have reconstructed the modified gravity in the framework of the scalar field models of dark energy and holographic Ricci dark energy (HRDE), a generalized version of the holographic dark energy presented in [2]. The viscous tachyon and quintessence scalar fields has been studied. Finally, $f(T)$ gravity is reconstructed in the presence of HRDE and a transition of the effective equation of state parameter from quintessence to phantom has been observed.

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ON SOME ELLIPTIC PROBLEMS WITH IRREGULAR BOUNDARY DATA

For given $q, \varkappa \in \mathbb{N}$, we consider a differential elliptic problem

$$Au = f \text{ in } \Omega, \quad B_j u + \sum\nolimits_{k=1}^{\varkappa} C_{j,k} v_k = g_j \text{ on } \Gamma, \;\; j = 1, ..., q + \varkappa. \tag{1}$$

Here, $\Omega \subset \mathbb{R}^n$ is a bounded domain with $\Gamma := \partial\Omega \in C^\infty$, and A, B_j, and $C_{j,k}$ are linear PDOs whose complex coefficients belong to $C^\infty(\overline{\Omega})$ (as for A) and to $C^\infty(\Gamma)$. We suppose $\operatorname{ord} A = 2q$, $\operatorname{ord} B_j \leq m_j$, and $\operatorname{ord} C_{j,k} \leq m_j + r_k$ for some $m_j, r_k \in \mathbb{Z}$. The distributions $u \in \mathcal{S}'(\Omega)$ and $v_k \in \mathcal{D}'(\Gamma)$ are unknown in (1). We assume $g_j \in \mathcal{D}'(\Gamma)$ and $f \in L_2(\Omega)$ and study local (up to Γ) properties of the solution $(u, v) := (u, v_1, \ldots, v_\varkappa)$ to problem (1) in the Hilbert Sobolev spaces $H^l(\Omega)$ and $H^l(\Gamma)$, with $l \in \mathbb{R}$.

Let an open set $U \subset \mathbb{R}^n$ satisfies $\Omega_0 := \Omega \cap U \neq \emptyset$ and $\Gamma_0 := \Gamma \cap U \neq \emptyset$. Then $H^l_{\mathrm{loc}}(\Omega_0, \Gamma_0)$ consists of all $u \in \mathcal{S}'(\Omega)$ such that $\chi u \in H^l(\Omega)$ for every $\chi \in C^\infty(\overline{\Omega})$ with $\operatorname{supp} \chi \subset \Omega_0 \cup \Gamma_0$; $H^l_{\mathrm{loc}}(\Gamma_0)$ is defined analogously.

Theorem 1. *Let $s < 2q$. Assume that $(u, v) \in \mathcal{S}'(\Omega) \times (\mathcal{D}'(\Gamma))^\varkappa$ is a solution to the problem* (1) *where $f \in L_2(\Omega)$ and all $g_j \in H^{s-m_j-1/2}_{\mathrm{loc}}(\Gamma_0)$. Then $u \in H^s_{\mathrm{loc}}(\Omega_0, \Gamma_0)$ and every $v_k \in H^{s+r_k-1/2}_{\mathrm{loc}}(\Gamma_0)$.*

Let $\|\cdot\|'_s$ and $\|\cdot\|''_{0,s}$ stand for the norms in the Hilbert spaces $H^s(\Omega) \oplus \bigoplus_{k=1}^{\varkappa} H^{s+r_k-1/2}(\Gamma)$ and $L_2(\Omega) \oplus \bigoplus_{j=1}^{q+\varkappa} H^{s-m_j-1/2}(\Gamma)$, resp.

Theorem 2. *Assume that (u, v) satisfies the hypothesis of Theorem* 1 *for some $s < 2q$. Let $\lambda > 0$, and let $\chi, \eta \in C^\infty(\overline{\Omega})$ satisfy* $\operatorname{supp} \chi \subset \operatorname{supp} \eta \subset \Omega_0 \cup \Gamma_0$ *and $\eta = 1$ in a neighbourhood of* $\operatorname{supp} \chi$. *Then*

$$\|\chi(u, v)\|'_s \leq c \left(\|\eta(f, g)\|''_{0,s} + \|\eta(u, v)\|'_{s-\lambda} \right)$$

for some number $c > 0$ not depending on u, v, f, and $g := (g_1, \ldots, g_{q+\varkappa})$.

These results are also true for a refined Sobolev scale and obtained together with A. Murach [1].

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SOME PROPERTIES OF THE SPECTRUM OF THE ALGEBRA OF BOUNDED TYPE SYMMETRIC ANALYTIC FUNCTIONS

By a symmetric function on ℓ_1 we mean a function which is invariant under any reordering of the sequence in ℓ_1. In the talk we consider an algebra of symmetric analytic functions on ℓ_1, which are bounded on bounded subsets. We investigate the spectrum of the algebra, that is a set of all continuous complex-valued homomorphisms.

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THE FIRST BOUNDARY VALUE PROBLEM FOR EQUATION $\frac{\partial u}{\partial t} - \triangle u = |u|^q \varrho^\gamma(x)$ IN THE CLASS OF GENERALIZED FUNCTIONS

Let $n \in \mathbb{N}$, Ω is a bounded domain in $\mathbb{R}^n$ with closed frontier S of class C^∞, $Q = \Omega \times (0,T]$, $\Sigma = S \times (0,T]$, $0 < T < +\infty$;

$$\varrho(x,t) = \begin{cases} \varrho_1(x) & \text{at } d(x) \to 0, \\ \sqrt{\varrho_2(t)} & \text{at } t \to 0, \\ 1, & \text{inside of the domain } Q, \end{cases}$$

where $\varrho(x) \equiv \varrho_1(x)$, $x \in \overline{\Omega}$, is a infinitely differentiable nonnegative function, which is a positive function on Ω, has the order of the distance $d(x)$ from the point x to S near S and $\varrho_1(x) \leq 1$, $x \in \overline{\Omega}$;
$\varrho_2(t)$, $t \in (0,T]$, is a infinitely differentiable nonnegative function, which is a positive function at $t \in (0,T]$, has the order t when $t \to 0$ and $\varrho_2(t) \leq 1$, $t \in (0,T]$; $0 \leq \varrho(x,t) \leq 1$, $(x,t) \in \overline{Q}$.

Let $D(\overline{\Sigma}) = C^\infty(\overline{\Sigma})$, $D(\overline{\Omega}) = C^\infty(\overline{\Omega})$;
$D^0(\overline{\Sigma}) = \{\varphi \in D(\overline{\Sigma}) : \ \frac{\partial^m}{\partial t^m}\varphi \,|_{\,t=T} = 0, \ m = 0, 1, \dots\}$,
$D_0(\overline{\Omega}) = \{\varphi \in D(\overline{\Omega}) : \varphi|_S = 0\}$.

The strokes will denote the spaces of linear continuous functionals on the respective functional spaces.

We introduce a functional space
$\mathcal{M}_k(Q) = \{\mathrm{v} \in L^1_{loc}(Q) : ||\mathrm{v}||_k = \int\limits_Q \varrho^k(x,t)|\mathrm{v}(x,t)|\, dxdt < +\infty\}$, $k \in \mathbb{R}$.

We study the problem

$$\frac{\partial u(x,t)}{\partial t} - \triangle u(x,t) = |u(x,t)|^q \varrho^\gamma(x), \ (x,t) \in Q,$$

$$u \,|_{\,\Sigma} = F_1(x,t), \ (x,t) \in \Sigma, \qquad u \,|_{\,t=0} = F_2(x), \ x \in \Omega,$$

$q \in (0,1)$, $\gamma \in (-1;0)$, $F_1 \in (D^0(\overline{\Sigma}))'$, $F_2 \in (D_0(\overline{\Omega}))'$.

Using the Schauder's method, there was obtained the sufficient conditions for solvability of this problem in the space $\mathcal{M}_k(Q)$.

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SUPERSYMMETRIC POLYNOMIALS, MULTISETS, AND COMPLEX DYNAMICS

Let $\mathbf{R}_n = \{\alpha_1, \ldots, \alpha_n\}$, $\alpha_1 = 1$ be the group of roots of unity and $\ell_1^{(n)}$ the Cartesian product of n copies of ℓ_1. Each element of $\ell_1^{(n)}$ can be written $x = (x^{(1)}, \ldots, x^{(n)})$, $x^{(j)} = (x_1^{(j)}, \ldots, x_m^{(j)}, \ldots) \in \ell_1$. On every copy of ℓ_1 we define polynomials

$$F_k^{(j)}\big(x^{(j)}\big) = \sum_{i=1}^{\infty} \big(x^{(j)}\big)^k, \quad k \in \mathbb{N}.$$

Let us consider the following relation of equivalence on $\ell_1^{(n)}$:

$$x \sim y \Leftrightarrow T_k(x) = T_k(y), \quad k \in \mathbb{N},$$

where

$$T_k(x) = \sum_{j=1}^{n} \alpha_j F_k^{(j)}\big(x^{(j)}\big).$$

Let us denote by $\mathcal{M}\mathbf{R}_n$ the quotient set of $\ell_1^{(n)}$ with respect to the relation " $\sim''$. The cases of $\mathcal{M}\mathbf{R}_1$ and $\mathcal{M}\mathbf{R}_2$ were considered in [1] and their application for complex dynamics in [2]. In he talk we will consider the general case and will show that $\mathcal{M}\mathbf{R}_n$ is a complete metric nonlinear ring with respect to some natural algebraic operations. In addition, we will consider the case when all coordinates of vectors $x^{(j)}$ are nonnegative integers and discuss some applications in Number Theory.

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DECOMPOSITION OF FUNCTIONS OF SMALL EXPONENTIAL TYPE IN PALEY - WIENER SPACES

The Paley - Wiener space W^p_σ, $\sigma > 0$, is the space of entire function f of exponential type $\le \sigma$ belonging to $L^p(\mathbb{R})$.

V. M. Dilnyi and T. I. Hishchak considered the following decomposition problem [1].

Problem 1.*Does each function $f \in W^p_\sigma$, $1 \le p \le 2$, admit the decomposition $f = \chi - \mu$, where χ and μ is analytical in $\mathbb{C}_+ = \{z : Rez > 0\}$ and $\chi \in E^1[\mathbb{C}(0; \frac{\pi}{2})]$, $\mu \in E^1[\mathbb{C}(-\frac{\pi}{2}; 0)]$.*

Theorem A. [2] *Let $f \in W^1_\sigma$. The functions χ and μ are a solution of the Problem 1 if and only if both of the following conditions are fulfilled*

$$\sum_{m=1}^{+\infty} \left| \sum_{k=-\infty}^{+\infty} \frac{c_k k}{\left(m - \frac{i}{2} - k\right)\left(m - \frac{i}{2} - ik\right)} \right| < +\infty, \tag{1}$$

$$\sum_{m=1}^{+\infty} \left| \sum_{k=-\infty}^{+\infty} \frac{c_k k}{\left(m + \frac{i}{2} + ik\right)\left(m + \frac{i}{2} - k\right)} \right| < +\infty. \tag{2}$$

We offer a solving the decomposition problem for an entire function of any small exponential type in $\mathbb{C}_- = \{z : \Re z < 0\}$.

Problem 2.*Does each function $f \in W^p_\sigma$, $1 \le p \le 2$, admit the decomposition $f = \hat{\chi} - \hat{\mu}$, where χ and μ are entire functions of any small exponential type $\alpha < 0$ in $\mathbb{C}_-$.*

The main result.

Theorem 1. *Let $f \in W^1_\sigma$. Functions $\hat{\chi}$, $\hat{\mu}$ are the solutions of the Problem 2 if and only if conditions (1) and (2) are valid.*

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ANALYTICAL ESTIMATES OF SPECTRAL APPROXIMATIONS OF SOME CLASSES OF DEGENERATE ELLIPTIC OPERATORS

We investigate a spectral approximation problem for some classes of degenerate elliptic differential operators. Namely, we consider the strongly degenerate elliptic differential operators in the Lebesgue space $L_q(\Omega)$ on a bounded domain Ω [1]. Such elliptic operators are characterized in a strong degeneration of the coefficients near the boundary. Their spectrum consists of isolated eigenvalues of finite algebraic multiplicity and the linear span of the associated eigenvectors is dense in $L_q(\Omega)$. Moreover, we consider the ordinary degenerate elliptic differential operators in a bounded interval $\Omega = (a, b)$ [2]. Such operators are self-adjoint with a discrete spectrum in $L_2(\Omega)$.

We prove the direct and inverse theorems that give precise estimates of approximation errors and which are connected with appropriate estimations by Bernstein-Jackson type inequalities. For this we use the quasi-normalized Besov-type approximation spaces, determined by the functional $E\,(t, u)$, which characterizes the shortest distance from an arbitrary function $u \in L_q(\Omega)$ to the closed linear span of spectral subspaces of the given elliptic operator. The approximation functional $E\,(t, u)$ in the Bernstein-Jackson type inequalities plays a similar role as the modulus of smoothness in the functions theory.

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DIRECT AND INVERSE SPECTRAL PROBLEMS FOR SINGULAR RANK-ONE PERTURBATION OF A SELF-ADJOINT OPERATOR

Assume A is a self-adjoint operator with discrete spectrum acting in a Hilbert space H. Denote by $H_{1/2}$ the domain of the quadratic form generated by A and by $H_{-1/2}$ its dual space. Let $B = A + \langle \cdot, \varphi \rangle \psi$ be a rank one perturbation of the operator A, where $\langle \cdot, \cdot \rangle$ denotes the pairing between the spaces $H_{1/2}$ and $H_{-1/2}$ and φ and ψ are elements of $H_{-1/2}$. In the talk, we study the spectral properties of the operator B; in particular, the asymptotics of eigenvalues μ_n of the operator B is found and a full description of the possible spectra of B is given. The possibility of reconstructing the Fourier coefficients of φ and ψ from the spectra of the operators A and B is studied.

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PROPERTIES OF INTEGRALS OF THE TYPE OF DERIVATIVES OF VOLUME POTENTIAL FOR ONE CLASS OF DEGENERATE $\vec{2b}$-PARABOLIC EQUATIONS

Let n_1, n_2, n_3 be given positive integer numbers such that $1 \le n_3 \le n_2 \le n_1$, $n := n_1 + n_2 + n_3$; $x := (x_1, x_2, x_3) \in \mathbb{R}^n$, $x_l := (x_{l1}, ..., x_{ln_j}) \in \mathbb{R}^{n_l}$, $l \in \{1, 2, 3\}$; $T > 0$. For some numbers b_1, ..., b_{n_1} from $\mathbb{N}$ we denote by $\vec{2b}$ the vector $(2b_1, ..., 2b_{n_1})$, by b the least common multiple of the numbers b_1, ..., b_{n_1}, by m_j a number b/b_j, $j \in \{1, ..., n_1\}$. If $k_1 := (k_{11}, ..., k_{1n_1}) \in \mathbb{Z}_+^{n_1}$ is a n_1-dimensional index, than $\|k_1\| := \sum\limits_{j=1}^{n_1} m_j k_{1j}$.

We considered integrals of the type

$$v(t,x) := \int\limits_0^t d\tau \int\limits_{\mathbb{R}^n} M(t,x;\tau,\xi) f(\tau,\xi) d\xi, \quad (t,x) \in (0,T] \times \mathbb{R}^n.$$

The kernel M is a complex-valued function and it has properties of the derivatives of the fundamental solution of the Cauchy problem (FSCP) for the equation

$$\begin{gathered}(\partial_t - \sum_{j=1}^{n_2} x_{1j}\partial_{x_{2j}} - \sum_{j=1}^{n_3} x_{2j}\partial_{x_{3j}} - A(t,\partial_{x_1}))u(t,x) = f(t,x), \\ (t,x) \in \Pi_{(0,T]}, \quad A(t,\partial_{x_1}) := \sum_{\|k_1\| \le 2b} a_{k_1}(t) \partial_{x_1}^{k_1}, \qquad (1)\end{gathered}$$

where coefficients a_{k_1} are continuous on $[0,T]$ functions and differential expression $\partial_t - A(t, \partial_{x_1})$ is uniformly $\vec{2b}$-parabolic on $[0,T] \times \mathbb{R}^{n_1}$.

The equation (1) is a degenerate equation of Kolmogorov type with $\vec{2b}$-parabolic part with respect to main variables.

To formulate the results we considered special weight Hölder spaces of functions which in certain way unlimited increase as $|x| \to \infty$. The obtained properties of the integral are described by fact that the function v belongs to these spaces. The results might be applied to establish the well-posedness of the Cauchy problem for the equations (1) and to construct FSCP for equations of the type (1) with coefficients dependent on space variables.

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EQUIVALENCE OF MATRICES IN THE RING OF THE BLOCK TRIANGULAR MATRICES

Let R be a principal ideal domain. Further, $M(n,R)$ and $GL(n,R)$ denote the ring of $n\times n$ matrices and the group of invertible $n\times n$ matrices over R, respectively, $BT(n_1,\dots,n_k,R)$ denotes the ring of the block upper triangular matrices $T=\mathrm{triang}(T_{11},\dots,T_{kk})=[T_{ij}]_1^k$, where $T_{ij}=0$ if $i>j$, $T_{ii}\in M(n_i,R)$, $i=1,\dots,k$. Block matrices, in particular, the block triangular matrices, are used in various fields of mathematics, such as stability theory [1], factorizations of matrices [2] and many others. Relevant is also the problem of establishing equivalences conditions of different types of such matrices [3].

We establish the conditions of the equivalence of matrices in the ring of the block triangular matrices. Block triangular matrices A and B are called equivalent in the ring $BT(n_1,\dots,n_k,R)$ if there exist invertible block triangular matrices P, $Q\in GL(n,BT)$ such that $PAQ=B$. Note that the equivalent block triangular matrices over R may are not equivalent in the ring $BT(n_1,\dots,n_k,R)$. The necessary condition of the equivalence of matrices in the ring $BT(n_1,\dots,n_k,R)$ is the equivalence of their corresponding diagonal blocks, but this condition is not sufficient.

Theorem. *Let the matrices A, $B\in BT(n_1,\dots,n_k,R)$ and let their corresponding diagonal blocks be equivalent, i.e., there exist P_i, $Q_i\in GL(n_i,R)$ such that $P_iA_{ii}Q_i=B_{ii}$ for all $i=1,\dots,k$.*

If $(\det A_{ii},\det A_{i+j,i+j})=1$ for all $i=1,\dots,k-1$, $j=1,\dots,k-i$, then the matrices A and B are equivalent in the ring $BT(n_1,\dots,n_k,R)$.

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A COUNTEREXAMPLE IN COCONVEX APPROXIMATION OF PERIODIC FUNCTIONS

Let 2π-periodic function $f \in \mathbb{C}$ change its convexity finitely even many times, in the period. We are interested in estimating the degree of approximation of f by trigonometric polynomials which are coconvex with it, namely, polynomials that change their convexity exactly at the points where f does. We list established Jackson-type estimates of such approximation where the constants involved depend on the location of the points of change of convexity and show that this dependence is essential by constructing a counterexample [1].

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ON CLASSIFICATION OF SYMMETRY REDUCTIONS FOR SOME $P(1,4)$-INVARIANT PARTIAL DIFFERENTIAL EQUATIONS

We plan to present some of the results concerning classifications of symmetry reductions for the following $P(1,4)$-invariant PDEs in the space $M(1,3) \times R(u)$:

– the Eikonal equation,

– the Euler-Lagrange-Born-Infeld equation,

– the Monge-Ampère equation.

Here, $M(1,3)$ is (1+3)-dimensional Minkowski space; $R(u)$ is the real number axis of the depended variable u.

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ON SYMMETRY REDUCTION AND INVARIANT SOLUTIONS OF THE EULER-LAGRANGE-BORN-INFELD EQUATION

Let us consider the Euler-Lagrange-Born-Infeld equation in the space $M(1,3) \times R(u)$

$$\Box u \,(1 - u_\nu u^\nu) + u^\mu u^\nu u_{\mu\nu} = 0,$$

where $u = u(x), x = (x_0, x_1, x_2, x_3) \in M(1,3)$, $u_\mu \equiv \dfrac{\partial u}{\partial x^\mu}$, $u_{\mu\nu} \equiv \dfrac{\partial^2 u}{\partial x^\mu \partial x^\nu}$, $u^\mu = g^{\mu\nu} u_\nu$, $g_{\mu\nu} = (1, -1, -1, -1)\delta_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$, $\Box$ is the d'Alembert operator.

Here, $M(1,3)$ is (1+3)-dimensional Minkowski space; $R(u)$ is the real number axis of the depended variable u.

I plan to present some of the results obtained concerning symmetry reduction of the equation under investigation.

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NUMERICAL INTEGRATION OF THE MATHISSON-PAPAPETROU EQUATIONS

The Mathisson-Papapetrou (MP) equations for Schwarzschild's metric are considered in the case when a spinning particle is moving in the equatorial plane and its spin is orthogonal to this plane. It is shown that for the case of ultrarelativistic motion the energies of particles with spin and without spin differ significantly under the same initial conditions. Because the system of equations is stiff, a comparison of the solutions of the system of MP equations for the Rosenbrock, Adams and the BDF methods is made.

A set of numerical calculations is performed to estimate the motion of the eigenvalue center of the spin fraction. In particular, according to these calculations, Table 1 illustrates the differences of the energies of spinning and spinless particles for some values of the initial tangential velocity $u_{\perp}(0)$ in the case of the fixed value of the particle's spin. The contribution of the spin-gravity coupling to the energy of a spinning particle becomes large when its velocity is highly relativistic.

$u_{\perp}(0)$	$E_{\text{spin}}/E_{\text{geod}}$
0	1
5.88	1.06
11.75	1.24
17.67	1.63
23.50	2.00

Table 1: On comparison of the energies of the spinning and spinless particles at different orbital velocity for $u_{\perp}(0) > 0$.

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INFINITE SERIES INVOLVING FIBONACCI (LUCAS) NUMBERS AND THE RIEMANN ZETA FUNCTION

Our note is devoted to combine very popular and important mathematical objects: the Riemann zeta function and Fibonacci (Lucas) numbers. Let F_n denote the n-th Fibonacci number and L_n the n-th Lucas number, both satisfying the recurrence $w_n = w_{n-1} + w_{n-2}$, $n \geq 2$, but with the initial conditions $F_0 = 0$, $F_1 = 1$ and $L_0 = 2$, $L_1 = 1$.

The Riemann zeta function is defined by

$$\zeta(s) = \sum\nolimits_{k\geq 1} \frac{1}{k^s}, \quad \Re(s) > 1.$$

Theorem 1. *For $m \geq 0$,*

$$\sum\nolimits_{k\geq 1} (\zeta(2k) - 1) F_{2k+m-1} = \frac{\pi}{2\sqrt{5}} \tan\frac{\sqrt{5}\pi}{2} L_m + \frac{F_{m+2}}{2},$$

$$\sum\nolimits_{k\geq 1} (\zeta(2k) - 1) L_{2k+m-1} = \frac{\sqrt{5}\pi}{2} \tan\frac{\sqrt{5}\pi}{2} F_m + \frac{L_{m+2}}{2}.$$

Theorem 2. *For $m \geq 0$,*

$$\sum_{k\geq 2} (\zeta(k) - 1) F_{k+m-1} = F_{m+1} + \frac{\pi F_{m-1}}{\sqrt{5}} \tan\frac{\sqrt{5}\pi}{2} + F_m \sum_{n=1}^{\infty} \frac{1}{n(n^2+n-1)},$$

$$\sum_{k\geq 2} (\zeta(k) - 1) L_{k+m-1} = L_{m+1} + \frac{\pi L_{m-1}}{\sqrt{5}} \tan\frac{\sqrt{5}\pi}{2} + L_m \sum_{n=1}^{\infty} \frac{1}{n(n^2+n-1)}.$$

Theorem 3. *For $m \geq 0$,*

$$\sum\nolimits_{k\geq 1} (\zeta(2k) - 1) \frac{F_{2k+m-1}}{k} = F_{m-1} \ln\Big(-\pi \sec\frac{\sqrt{5}\pi}{2}\Big) + \frac{2L_{m-1}\ln\alpha}{\sqrt{5}},$$

$$\sum\nolimits_{k\geq 1} (\zeta(2k) - 1) \frac{L_{2k+m-1}}{k} = L_{m-1} \ln\Big(-\pi \sec\frac{\sqrt{5}\pi}{2}\Big) + 2\sqrt{5} F_{m-1} \ln\alpha,$$

where $\alpha = (1+\sqrt{5})/2$.

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ON ADEQUACY OF FULL MATRICES OVER ADEQUATE RINGS

The notion of an adequate domain was originally defined by Helmer. A ring R is adequate if R is a commutative Bezout domain and for every $a \neq 0$, and b in R we can write $a = cd$, with $(c,\ b) = 1$, and $(d_i, b) \neq 1$ for every nonunit divisor d_i of d. By definition, every adequate domain is a Prufer domain. So, every principal ideal domain is adequate. An example of an adequate ring which is not a principal ideal domain is furnished by the set of integral functions with coefficients in a field. Also, it is clear to see that a local ring is adequate.

The adequate rings with zero-divisors in Jacobson radical were studied by Kaplansky. Gillman and Henriksen have shown that a von Neumann regular ring is adequate. The first example of Bezout nonadequate domain, which is an elementary divisors domain, was constructed by Henriksen. Gatalevych was the first, who studied noncommutative adequate rings and their generalizations. He has proved that the generalized right adequate duo-Bezout domain is an elementary divisor domain.

Theorem 1. *Let R be an adequate ring and $A = P_A^{-1}\operatorname{diag}(\alpha_1,\ \alpha_2)Q_A^{-1}$, $\alpha_1|\alpha_2$, $B = P_B^{-1}\operatorname{diag}(1,\ \beta_2)Q_B^{-1}$, $\beta_2 \neq 0$ are matrices over R, and $AR + BR \neq R$. Then the matrix B can be represented as the product: $B = ST$, where $AR + TR = R$ and for each nontrivial left divisor S' of the matrix S the condition $AR + S'R \neq R$ is satisfied.*

Thus, it makes sense to introduce the following definition.

Definition. Element a of the ring R is called right adequate if for an arbitrary element $b \in R$ there exist such elements $r, s \in R$ that $a = r \cdot s$, where $rR + bR = R$, and for an arbitrary element $s' \in R$ such that $sR \subset s'R \neq R$, we get $s'R + bR \neq R$.

So Theorem 1 can be reformulated as follows.

Theorem 2. *Full nonsingular second-order matrices over the adequate ring R are right adequate elements of the ring $M_2(R)$.*

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ON MULTI-INTERVAL STURM-LIOUVILLE PROBLEMS WITH DISTRIBUTIONAL COEFFICIENTS

We investigate spectral properties of Sturm-Liouville operators in the direct sum of spaces $\oplus_{k=1}^{m} L_2\left([a_{k-1}, a_k], \mathbb{C}\right)$ under minimal conditions for the regularity of the coefficients. That is, on every interval the formal Sturm-Liouville differential expression

$$l_k(y) = -(p_k(t)y')' + q_k(t)y + i((r_k(t)y)' + r_k(t)y')$$

is given with coefficients p_k, q_k and r_k which satisfy the conditions

$$q_k = Q_k', \left\{1/\sqrt{|p_k|}, Q_k/\sqrt{|p_k|}, r_k/\sqrt{|p_k|}\right\} \subset L_2\left([a_{k-1}, a_k], \mathbb{R}\right),$$

where the derivatives Q_k' are understood in the sense of distributions. These expressions similarly to [1] may be correctly defined as Shin-Zettl quasi-differential expressions, which generate minimal and maximal operators $L_{k,0}$ and $L_{k,1}$. Then we consider maximal and minimal operators $L_{\max} = \oplus_{k=1}^{m} L_{k,1}$ and $L_{\min} = \oplus_{k=1}^{m} L_{k,0}$.

For such operators we give constructive descriptions of self-adjoint, maximal dissipative and maximal accumulative extensions and also generalized resolvents in terms of homogeneous boundary conditions. Proof applies the machinery of boundary triplets of symmetric differential operators and regularization of the formal Sturm-Liouville expression with Shin-Zettl quasi-derivatives.

These results are new for one-interval boundary value problems as well. Their exact statement and proof may be found in [2].

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VARIOUS TYPES OF NUMERICAL METHODS FOR FINDING THE APPROXIMATE SOLUTION OF FRACTIONAL DIFFERENTIAL EQUATIONS

The objective of this paper is to find the approximate solution of Fractional Burgers-Huxley Equation (FBHE) is using the homotopy perturbation method (HPM). The Differential Transform Method (DTM) is applied to drive its solution (approximate) of the fractional Bratu-type equations and estimate the absolute error. The rapid convergence and uniqueness of both techniques are also studied through different graphical representations. The fractional integral of order α can be defined as Riemann improper integral (J_α). We have used notations $D^{n\alpha}$ for Jumarie type fractional derivative operator here $n \in \mathfrak{R}, \alpha \in (0, 1]$. In this paper, many figures at different values of alpha demonstrate that when the estimation of α is increment, $w(x,t)$ increment and when alpha close to zero, then $w(x,t)$ will be decrease. The HPM has been successfully applied to find an approximate solution of the FBHE and some special cases of the equation. This technique produces the same solution as the HPM with the proper choice of initial supposition. The HPM has been successfully applied to find an approximate solution of the FBHE and some special cases of the equation. Three examples illustrate to prove that the presented techniques efficiency and implementation of the technique and the results are equivalence with true (exact) solutions and discuss its absolute error. It shows that the present technique is powerful for finding the numerical solution of FBHE using Mathematica.

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DIOPHANTINE APPROXIMATION OF ZERO BY VALUES OF REDUCIBLE POLYNOMIALS

Most of the results obtained by the Minsk school of Diophantine approximation describe approximation of real or complex numbers using irreducible polynomials or roots thereof. However, in recent years there has been an uptick of interest in approximations that involve reducible polynomials. It turns out that this aspect of Diophantine approximation requires extensive additional studies. Until recently, the primary approach to studying approximation properties of reducible polynomials has been the use of Gelfond's Lemma. Another well-studied approach is described in the book "Metric Diophantine approximation on manifolds" [1] by V. Bernik and M. Dodson.

The authors present an improvement of Bernik and Dodson's results. Let μA denote the Lebesgue measure of a measurable set $A \subset \mathbb{R}$. Let $\mathcal{R}_n(Q)$ be the set of reducible polynomials $P(x) = \sum_{i=0}^{n} a_i x^n$ of degree n and height Q.

Theorem. *The inequality* $|P(x)| < Q^{-(n-1)-\varepsilon}$ *is satisfied in reducible polynomials* $P(x) = \prod_{j=1}^{k} t_j(x)$, $2 \leq k < n$ *only if* $x \in B_2$, *where* $\mu B_2 < Q^{-\varepsilon_1}$, $0 < \varepsilon_1 < \varepsilon$. $\#\mathcal{R}_n(Q) \ll Q^{n-k+2}$.

In conclusion, note that the results obtained by the authors have applications to the problem of small denominators in equations of mathematical physics [2].

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ON THE ISOMORPHISM AND SOME OTHER PROPERTIES OF THE FRÉCHET ALGEBRAS OF ENTIRE FUNCTIONS OF BOUNDED TYPE ON BANACH SPACES

We consider the Fréchet algebras of entire functions of bounded type, generated by countable sets of algebraically independent homogeneous polynomials on complex Banach spaces. We investigate the properties of such algebras. In particular, we prove that any two of such algebras, generated by homogeneous polynomials of norm 1 and which spectra are sets of point-evaluation functionals at points of corresponding spaces, are isomorphic.

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THE CENTRALLY EXTENDED LIE ALGEBRA OF FRACTAL INTEGRAL-DIFFERENTIAL OPERATORS AND INTEGRABLE FRACTAL DYNAMICAL SYSTEMS

The Lie algebra $\mathbb{A}_\alpha := \mathbb{A}_0\{\{D^\alpha, D^{-\alpha}\}\}$, consisting of the fractal integral-differential operators $\mathrm{a}_\alpha := \sum_{j\in\mathbb{Z}_+} a_j D^{\alpha(m_\alpha - j)}$, where $\mathbb{A}_0 := A\{\{D, D^{-1}\}\}$ is the Lie algebra of integral-differential operators, $A := W_2^\infty(\mathbb{R};\mathbb{C}) \cap W_\infty^\infty(\mathbb{R};\mathbb{C})$, D^α is a fractional derivative, $\alpha \in \mathbb{C}$, $\operatorname{Re}\alpha \neq 0$, $m_\alpha \in \mathbb{Z}_+$ and $a_j \in \mathbb{A}_0$, $j \in \mathbb{Z}_+$, is considered. This Lie algebra allows the splitting into the direct sum of its Lie subalgebras $\mathbb{A}_\alpha = \mathbb{A}_{\alpha,+} \oplus \mathbb{A}_{\alpha,-}$, where $\mathbb{A}_{\alpha,+}$ is the Lie subalgebra of the formal power series by the operator D^α. One constructs the central extension $\hat{\mathbb{A}}_\alpha := \bar{\mathbb{A}}_\alpha \oplus \mathbb{C}$ of the Lie algebra $\bar{\mathbb{A}}_\alpha := \prod_{y\in\mathbb{S}^1} \mathbb{A}_\alpha$ by the Maurer-Cartan 2-cocycle $\omega_2(.,.)$ on $\bar{\mathbb{A}}_\alpha$, possessing the commutator:

$$[(\mathrm{a}_\alpha, d), (\mathrm{b}_\alpha, e)] = ([\mathrm{a}_\alpha, \mathrm{b}_\alpha], \omega_2(\mathrm{a}_\alpha, \mathrm{b}_\alpha)), \quad (\mathrm{a}_\alpha, d), (\mathrm{b}_\alpha, e) \in \hat{\mathbb{A}}_\alpha,$$

where $[\mathrm{a}_\alpha, \mathrm{b}_\alpha] = \mathrm{a}_\alpha \circ \mathrm{b}_\alpha - \mathrm{b}_\alpha \circ \mathrm{a}_\alpha$, $\omega_2(\mathrm{a}_\alpha, \mathrm{b}_\alpha) := \int_{\mathbb{S}^1} (\mathrm{a}_\alpha, \partial \mathrm{b}_\alpha/\partial y)dy$, $(\mathrm{a}_\alpha, \mathrm{b}_\alpha) := \int_{\mathbb{R}} \operatorname{res}_D (\operatorname{res}_{D_\alpha} (\mathrm{a}_\alpha \circ \mathrm{b}_\alpha D^{-\alpha}))dx$. In addition, $\operatorname{res}_{D_\alpha}$ denotes a coefficient at $D^{-\alpha}$ for any fractal integral-differential operator as well as res_D denotes a coefficient at D^{-1} for any integral-differential operator.

The standard Lie-Poisson bracket, deformed by the space endomorphism $\mathcal{R} = (P_+ - P_-)/2$ ($P_\pm$ are projectors on $\mathbb{A}_{\alpha,\pm}$ respectively), is proven to generate the hierarchy of Lax type Hamiltonian flows:

$$\partial l_\alpha/\partial t_j = [P_+(\nabla\gamma_j(l_\alpha)), l_\alpha - c\partial/\partial y], \quad (l_\alpha, c) \in \hat{\mathbb{A}}_\alpha^*, \quad t_j \in \mathbb{R}, \ j \in \mathbb{N}, \quad (1)$$

where ∇ is a gradient operator, on the dual space $\hat{\mathbb{A}}_\alpha^* \simeq \hat{\mathbb{A}}_\alpha$ to $\hat{\mathbb{A}}_\alpha$ with respect to the scalar product such that $((\mathrm{a}_\alpha, d), (\mathrm{b}_\alpha, e)) = \int_{\mathbb{S}^1} (\mathrm{a}_\alpha, \mathrm{b}_\alpha)dy + ed$ for any $(\mathrm{a}_\alpha, d), (\mathrm{b}_\alpha, e) \in \hat{\mathbb{A}}_\alpha$ by means of the Casimir invariants $\gamma_j \in I(\hat{\mathbb{A}}_\alpha)$, $j \in \mathbb{N}$, as Hamiltonians. The hierarchy (1) is shown to be reduced to the integrable fractal analog of the Kadomtsev-Petviashvili equation hierarchy when $\tilde{l}_\alpha = D^{2\alpha} + vD^\alpha + u$, $u, v \in \bar{\mathbb{A}}_0$, and $c = 1$.

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ENTIRE ANALYTIC FUNCTIONS OF UNBOUNDED TYPE ON BANACH SPACES

Let X be an infinite dimensional complex Banach space. The Banach space of all continuous n-homogeneous polynomials on X is denoted by $\mathcal{P}(^n X)$.

The space of all entire analytic functions $f\colon X \to \mathbb{C}$ on X is denoted by $H(X)$. If radius of boundedness is ∞, then f is bounded on all bounded subsets of X.The set of all functions of bounded type on X is denoted by $H_b(X)$. We say that a function f is an entire function of *unbounded type* if $f \in H(X) \setminus H_b(X)$.

Theorem. *Let us suppose that there is a dense subset $\Omega \subset X$ and a sequence of polynomials $P_n \in \mathcal{P}(^n X)$, $\limsup_{n\to\infty} \|P_n\|^{1/n} = c$, $0 < c < \infty$ such that for every $z \in \Omega$ there exists $m \in \mathbb{N}$ such that for every $y \in X$,*

$$B_{P_n}(\underbrace{z, \dots, z}_{k}, \underbrace{y, \dots, y}_{n-k}) = 0$$

for all $k > m$ and $n > k$. Then

$$g(x) = \sum_{n=1}^{\infty} P_n(x) \in H(X) \setminus H_b(X).$$

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APPROXIMATION OF APPELL FUNCTION $F_4(1,2;2,2,Z_1,Z_2)$ BY MULTIDIMENSIONAL C–FRACTION

The hypergeometric function method naturally provides the analytic expressions of scalar integrals corresponding to the Feynman diagrams in some connected regions of independent kinematic variables [1]. We consider Appell hypergeometric function F_4 defined by double power series [2]:

$$F_4(a,b;c,c';z_1,z_2) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b)_{m+n}}{(c)_m(c')_n} \frac{z_1^m z_2^n}{m!n!},$$

where parameters a, b, c, c' are complex numbers, z_1, z_2 are complex variables, c, c' are not equal to $0, -1, -2, \ldots$, $(d)_k = d(d+1)\ldots(d+k-1)$ is the Pochhammer symbol.

In order to approximate Appell function $F_4(1,2;2,2,z_1,z_2)$ we use the multidimensional C–fraction [3]

$$\cfrac{1}{1 - \cfrac{z_1}{1 - \cfrac{1/2z_2}{1- \ddots}} - \cfrac{z_2}{1 - \cfrac{z_1}{1 - \ddots}}}.$$

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INVERSE SCATTERING FOR REFLECTIONLESS SCHRÖDINGER OPERATORS AND GENERALIZED KDV SOLITONS

In this talk, we discuss Schrödinger operators $S_q := -\frac{d^2}{dx^2} + q$ on the line with real-valued integrable reflectionless potentials q.

In particular, we give a complete characterization of such operators in terms of their scattering data, sequences of eigenvalues and norming constants, and suggest an explicit formula producing all such potentials, thus completely solving the related direct and inverse scattering problems. Using the inverse scattering transform approach [1], we then describe all solutions of the Korteweg–de Vries (KdV) equation whose initial profile is an integrable reflectionless potential. Such solutions stay integrable and reflectionless for all $t \geq 0$ and can be called *generalized soliton solutions* of the KdV.

This research extends and specifies in several ways the previous work on reflectionless potentials [3,4] and generalized soliton solutions of the KdV equation [2,3].

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DETERMINATION OF THE MINOR COEFFICIENT IN A DEGENERATE PARABOLIC EQUATION

In a rectangle $Q_T = \{(x,t) : 0 < x < h, 0 < t < T\}$ we consider the coefficient inverse problem for determination of two time dependent functions $b_1 = b_1(t), b_2 = b_2(t)$ in the minor coefficient in the one-dimensional degenerate parabolic equation

$$u_t = a(t)t^{\beta}u_{xx} + (b_1(t)x + b_2(t))u_x + c(x,t)u + f(x,t) \qquad (1)$$

with initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \qquad (2)$$

boundary conditions

$$u_x(0,t) = \mu_1(t),\ u_x(h,t) = \mu_2(t),\ t \in [0,T], \qquad (3)$$

and heat moments as overdetermination conditions

$$\int\limits_0^h u(x,t)dx = \mu_3(t), \quad t \in [0,T], \qquad (4)$$

$$\int\limits_0^h xu(x,t)dx = \mu_4(t), \quad t \in [0,T]. \qquad (5)$$

It is known that $a(t) > 0, t \in [0,T]$. Applying the Schauder fixed point theorem there is established conditions of existence of the classical solution to the named problem. For this aim the Green function of the second boundary value problem for the heat equation and its properties are used.

We prove the uniqueness of the solution taking into account the properties of the solutions of the homogeneous integral equations of the second kind with integrable kernels.

The case of weak power degeneration is investigated $(0 < \beta < 1)$.

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THE TWO-POINT PROBLEM FOR THE PARTIAL DIFFERENTIAL EQUATION OF AN EVEN ORDER

In a cylindrical domain $D = [0,T] \times \Omega$, $T > 0$, Ω is a unit circle $(\mathbb{R}/2\pi\mathbb{Z})$, we study such two-point problem for the partial differential equation

$$L(\partial_t, \partial_x)u(t,x) \equiv \sum_{s_0+s_1 \leq 2n} a_{s_0,s_1} \partial_t^{s_0} \partial_x^{s_1} u = 0, \quad (t,x) \in D, \tag{1}$$

$$\partial_t^{j-1} u\big|_{t=0} = \varphi_j(x), \quad \partial_t^{j-1} u\big|_{t=T} = \psi_j(x), \quad x \in \Omega, \quad j = 1, \ldots, n, \tag{2}$$

where $\partial_t = \partial/\partial t$, $\partial_x = \partial/\partial x$, a_{s_0,s_1} are complex numbers, $a_{2n,0} = 1$.

We established the solvability of this problem in the spaces of exponential type on variable x, when the the real parts of the roots of the polynomial $\sum\limits_{j=0}^{2n} a_{j,2n-j}(\lambda/i)^j$ are different. Note that the problem of small denominators [1] is absent in considered case.

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NONLOCAL BOUNDARY-VALUE PROBLEM FOR PARTIAL DIFFERENTIAL EQUATION IN AN UNBOUNDED STRIP

Let H_α, $\alpha > 0$, is the classical Sobolev class of functions $\varphi(x) \in L_2(\mathbb{R})$, such that $(1+\xi^2)^{\alpha/2}\tilde{\varphi}(\xi) \in L_2(\mathbb{R})$, where $\tilde{\varphi}(\xi)$ is the Fourier transform of the function $\varphi(x)$: $\tilde{\varphi}(\xi)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\varphi(x)e^{-ix\xi}dx$. The norm in the space H_α is introduced by the equality $\|\varphi(x);H_\alpha\|^2=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|\tilde{\varphi}(\xi)|^2\left(1+\xi^2\right)^\alpha d\xi$. By $C^n([0,T],H_\alpha)$, $\alpha \geq n$, $n \in \mathbb{N}$, we denote a space of functions $u(t,x):\Pi(T)\to\mathbb{C}$, where $\Pi(T)=\{(t,x): t\in[0,T], x\in\mathbb{R}\}$, $T>0$, such their derivatives $\partial^r u(t,x)/\partial t^r$, $r=0,1,\ldots,n$, for any $t\in[0,T]$ belong to spaces $H_{\alpha-r}$ and are continuous in the variable on t in these spaces. The norm in the space $C^n([0,T],H_\alpha)$ is introduced by the equality $\|u(t,x);C^n([0,T],H_\alpha)\| = \sum_{r=0}^{n}\max_{t\in[0,T]}\|\partial^r u(t,x)/\partial t^r;H_{\alpha-r}\|$.

The report is devoted to the presentation of the results, which we obtained by studying the nonlocal boundary-value problem for high-order partial differential equation in an unbounded strip $\Pi(T)$:

$$\sum_{s_1+s_2\leq n} a_{s_1,s_2}\frac{\partial^{s_1+s_2}u(t,x)}{\partial t^{s_1}\partial x^{s_2}} = 0, \quad (t,x)\in\Pi(T),$$

$$u_t^{j-1}(0,x) - \mu_j u_t^{j-1}(T,x) = \varphi_j(x), \quad j=1,\ldots,n, \quad x\in\mathbb{R},$$

where $a_{s_1,s_2}\in\mathbb{C}$, $a_{n,0}=1$, $\mu_j\neq 0$, $j=1,\ldots,n$.

The conditions of well-posedness in the space $C^n([0,T],H_\alpha)$ of this problem are established in the case where the real parts of the roots of its characteristic equation are different and nonzero.

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PROPERTIES OF GREEN'S OPERATORS AND CONJUGATE OPERATORS GENERATED BY CAUCHY PROBLEM FOR EIDELMAN PARABOLIC SYSTEMS, THEIR APPLICATIONS

Cauchy problem for a parabolic in the sense of Eidelman system of first order with respect to the time variable t partial differential equations (in which the spatial variables are unequal) is considered. Cauchy problem for such systems in positive Hölder spaces of bounded functions and some fast-increasing functions is currently well studied in [1]. For this problem, conjugate Green's operators are constructed, and their bounded action is established in the positive Hölder spaces of specially selected decreasing functions. With the use of the norms of conjugate operators in such spaces we define the corresponding negative Hölder spaces and prove the unique solvability theorem for Cauchy problem in these spaces. Similar results for boundary-value problems for parabolic in the sense of Petrovskii systems were obtained in [2].

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GREEN'S MATRIX OF A GENERAL MODEL BOUNDARY-VALUE PROBLEM FOR THE EIDELMAN PARABOLIC SYSTEMS

The general model $\overrightarrow{2b}$-parabolic boundary-value problem is considered in the $\Pi_T^+ := \{(t,x) := (x_1,\ldots,x_n) \in \mathbb{R}^{n+1} |\ t \in (0,T],\ x_n > 0\}$. That means, we consider the Eidelman $\overrightarrow{2b}$-parabolic system of equations and boundary conditions of any orders at $x_n = 0$. Equations and boundary conditions have not in corresponding sense minor members and their coefficients are constants, and boundary expressions satisfy the complementarity condition.

The main results for such problem are obtained and ones are formulated as follows propositions:

1) the correct solvability in the anisotropic Hölder spaces $H^{2s+l+\lambda}(\bar{\Pi}_T^+)$ is established; these spaces are anisotropic with respect to all variables;

2) solutions from the spaces $H^{2s+l+\lambda}(\bar{\Pi}_T^+)$ have the integral form through the corresponding right-hand functions of the boundary-value problem; Green's matrix of the boundary-value problem consist of the kernels G_0, G_1, G_2 of this integral representation for the solution; all elements of Green's matrix are ordinary functions in the case when general order of boundary expressions less then the order of the system of equations, and elements of G_0 and G_2 have summands which are linear combination consist of derivatives of Dirac δ-function in the opposite case; the distributions are concentrated in $t = 0$ and $x_n = 0$;

3) the results of propositions 1) and 2) are generated for a case of the anisotropic Hölder spaces of functions, that have an exponential grows as $|x| \to \infty$.

Some of these results are contained in [1].

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ON A MULTIPOINT PROBLEM FOR THE PARTIAL DIFFERENTIAL EQUATION IN COMPLEX DOMAIN

We consider such multipoint problem

$$\frac{\partial^n u(t,z)}{\partial t^n} + \sum_{j=0}^{n-1} a_j\, A^{n-j}\left(\frac{\partial}{\partial z}\right)\frac{\partial^j u(t,z)}{\partial t^j} = 0, \quad t, z \in \mathbb{C}, \tag{1}$$

$$u(jt_0, z) = \varphi_j(z), \quad t_0 \in \mathbb{C}\backslash\{0\}, \quad j = 1,\ldots,n, \quad z \in \mathbb{C}, \tag{2}$$

where $a_j \in \mathbb{C}$, $j = 0,1,\ldots,n-1$, $A(z)$ is a polynomial [2, 3]. Denote by $\lambda_1,\ldots,\lambda_n$ the roots of the polynomial $L(\lambda,1) \equiv \lambda^n + \sum\limits_{j=0}^{n-1} a_j\lambda^j$. We obtain the next result about solvability of the problem (1), (2) in the space $\mathrm{Exp}(\Omega)$ [1].

Theorem 1. *Let $\Omega \subset \mathbb{C}$ be a Runge domain and for all $z \in \Omega$ and $k \in \mathbb{Z}$ the inequalities*

$$(\lambda_j - \lambda_q)A(z)t_0 \neq i2\pi k, \quad 1 \leq q < j \leq n,$$

are true. If $\varphi_j \in \mathrm{Exp}(\Omega)$, $j = 1,\ldots,n$, then exists the unique solution $u(t,z)$ of the problem (1), (2), *which is analytic on variable $t \in \mathbb{C}$ and belongs to* $\mathrm{Exp}(\Omega)$ *on variable z for every fixed $t \in \mathbb{C}$.*

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FINITE TIME STABILIZATION OF NONAUTONOMOUS FIRST ORDER HYPERBOLIC SYSTEMS

The talk is devoted to the phenomenon of finite time stabilization in initial boundary value problems for nonautonomous decoupled linear first-order one-dimensional hyperbolic systems. We establish sufficient and necessary conditions ensuring that solutions stabilize to zero in a finite time for any initial L^2-data. We give a combinatorial criterion stating that the stabilization occurs if and only if the matrix of reflection boundary coefficients corresponds to a directed acyclic graph. An equivalent algebraic criterion is that the adjacency matrix of this graph is nilpotent. In the case of autonomous hyperbolic systems we also provide a spectral stabilization criterion. Moreover, we analyse robustness properties of all these criteria.

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ALGEBRAIC BASIS OF THE ALGEBRA OF BLOCK-SYMMETRIC POLYNOMIALS ON THE SPACES $\ell_{p_1} \oplus \ldots \oplus \ell_{p_n}$

Let us denote by $\mathcal{X} = \ell_{p_1} \oplus \ldots \oplus \ell_{p_n}$ the space with elements $x = (x^1, \ldots, x^n)$ where $x^j = (x^j_1, x^j_2, \ldots, x^j_n, \ldots) \in \ell_{p_j}, j = 1, \ldots, n$.

The space $\mathcal{X}$ with norm

$$||x||_{\mathcal{X}} = ||x^1||_{\ell_{p_1}} + \ldots + ||x^n||_{\ell_{p_n}},$$

where

$$||x^i||_{\ell_{p_i}} = \left(\sum_{k=1}^{\infty} |x^i_k|^{p_i}\right)^{\frac{1}{p_i}}, i = 1, \ldots, n$$

is a Banach space.

A polynomial P on the space $\mathcal{X}$ is called block-symmetric (or vector-symmetric) if:

$$P\left(\begin{pmatrix} x_1^1 \\ \ldots \\ x_1^n \end{pmatrix}, \ldots, \begin{pmatrix} x_k^1 \\ \ldots \\ x_k^n \end{pmatrix}, \ldots\right) = P\left(\begin{pmatrix} x_1^1 \\ \ldots \\ x_1^n \end{pmatrix}_{\sigma}, \ldots, \begin{pmatrix} x_k^1 \\ \ldots \\ x_k^n \end{pmatrix}_{\sigma}, \ldots\right),$$

for every permutation σ on the set of natural numbers $\mathbb{N}$, where $\begin{pmatrix} x_i^1 \\ \ldots \\ x_i^n \end{pmatrix} \in \mathbb{C}^n$.

Let us denote by $\mathcal{P}_{vs}(\mathcal{X})$ the algebra of block-symmetric polynomials on $\mathcal{X}$.

In this talk we consider algebraic basis of the algebra of block-symmetric polynomials on $\mathcal{X} = \ell_{p_1} \oplus \ldots \oplus \ell_{p_n}$.

Theorem *The algebraic basis of the algebra $\mathcal{P}_{vs}(\mathcal{X})$ form polynomials*

$$H^{k_1, \ldots, k_n} = \sum_{i=1}^{\infty} (x_i^1)^{k_1} \ldots (x_i^n)^{k_n}$$

for all $k_j \geq 0, j = 1, \ldots, n$ such that $\frac{k_1}{p_1} + \ldots + \frac{k_n}{p_n} \geq 1$.

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MULTIDIMENSIONAL ASSOCIATED AND REGULAR C-FRACTIONS AND MULTIPLE POWER SERIES

One of the approaches to represent analytic functions by continued fractions is the construction of a corresponding continued fraction [1].

Consider an N-dimensional continued fraction

$$\overset{\infty}{\underset{i=0}{D}} \frac{a_{\underbrace{i,i,\ldots,i}_{N}}(z)}{F_i(z)},$$

where $F_i(z)$ is the sum of p-dimensional continued fractions $(1 \leq p \leq N-1)$ with approximants

$$f_n(z) = \overset{n}{\underset{i=0}{D}} \frac{a_{\underbrace{i,i,\ldots,i}_{N}}(z)}{F_i^{(n-1)}(z)},$$

where $F_i^{(n-1)}(z)$ is the sum of the $(n-1)$th approximants of p-dimensional continued fractions $(1 \leq p \leq N-1)$.

One of the possible algorithms for the development of a formal N-multiple power series into a functional N-dimensional regular C-fraction or associated continued fraction corresponding to this series is proposed. In this case, development algorithms into corresponding and two-dimensional corresponding fractions for the power and double power series are used [2].

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NONLOCAL PROBLEM WITH INTEGRAL CONDITIONS FOR EVOLUTION EQUATIONS OF HIGHER ORDER

Let B be a Banach space, $A : B \to B$ is a linear operator, for which arbitrary powers A^n, $n = 2, 3, ...$ are also defined in B, denote by $x(\lambda)$ the eigenvector of the operator A, which corresponds to its eigenvalue $\lambda \in \Lambda \subseteq \mathbb{C}$.

We consider the problem

$$\left[\frac{d^n}{dt^n} + \sum_{j=1}^{n} a_j\left(A\right)\frac{d^{n-j}}{dt^{n-j}}\right]U(t) = 0, \tag{1}$$

$$\left.\frac{d^kU}{dt^k}\right|_{t=0} + \left.\frac{d^kU}{dt^k}\right|_{t=T} + \int_0^T t^{k-1}U(t)dt = \varphi_k, \quad k = \{0, ..., n-1\}, \tag{2}$$

where $a_j(\lambda)$, $j = \{1, ..., n\}$, are analytic functions in Λ, $\varphi_k \in B$, $T > 0$.

Using the differential-symbol method [1], we construct a solution of the problem (1), (2).

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EFFECTIVE PARAMETERS OF BENDING WAVES IN PLATES WITH DISTRIBUTED NON-CONTRAST THIN INHOMOGENEINS

A method for studying the effective propagation velocities of bending waves in a thin elastic plate with an array of randomly distributed thin rectilinear noncontrast elastic inclusions of constant thickness is proposed. The motion of the plate is described according to Kirchhoff's hypotheses. Inclusions are in the plate in perfect contact conditions at steady-state oscillations. The model of elasto-dynamic interaction of a thin plate with inclusion was obtained using the methods of the singular perturbations theory [1]. Cases of aligned and randomly orientation of inclusions in the plate are considered. The averaged dynamic parameters of the composite were determined using Foldy's theory of homogenization [2, 3] with the involvement of the solution of the problem of bending wave scattering by the corresponding local obstacle. The influence of mechanical properties of composite components and filler concentration on the speed of bending wave propagation in it is analyzed.

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IMPLEMENTATION OF EXACT THREE-POINT DIFFERENCE SCHEME FOR STURM-LIOUVILLE PROBLEM

The exact three-point difference scheme (ETDS) on irregular grid for boundary-value problems for linear second-order ordinary differential equations is proposed in [3]. Moreover, an algorithm is given that uses truncated three-point difference schemes of a high order of accuracy for the implementation of the ETDS. However, the practical application of such schemes in the case of variable coefficients of the differential equation requires one to calculate multidimensional integrals in each grid node. In paper [2] was show that, for the determination of the coefficients and the right-hand side of the ETDS, it is necessary to solve four auxiliary initial value problems with smooth coefficients. In [1] the results [3] were transferred to the Sturm-Liouville problems.

In this research a new algorithmic implementation of exact three-point difference schemes on irregular grid for the Sturm-Liouville problem is developed. It is shown that to compute the coefficients of the exact scheme in an arbitrary grid node, it is necessary to solve two initial value problems for second order linear ordinary differential equations.

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A PROBLEM WITH INTEGRAL CONDITIONS FOR FACTORIZED PARTIAL DIFFERENTIAL EQUATIONS

In the domain $\mathcal{D}^p = \{(t,x) : t \in (0,T), x \in \mathbb{R}^p\}$ we consider the following problem

$$\prod_{j=1}^{n} \left(\frac{\partial}{\partial t} - \lambda_j A\left(\frac{\partial}{\partial x}\right)\right) u(t,x) = 0, \quad (t,x) \in \mathcal{D}^p, \tag{1}$$

$$\int_0^T t^{r_j} u(t,x)dt = \varphi_j(x), \quad j = 1,\dots,n, \quad x \in \mathbb{R}^p, \tag{2}$$

where $\lambda_j \in \mathbb{R}$, $r_j \in \mathbb{Z}_+$, $j = 1,\dots,n$; $\lambda_j \neq \lambda_q$, $r_j \neq r_q$, $j \neq q$; we suppose that a symbol of operator $A(\eta)$, $\eta \in \mathbb{R}^p$, satisfy inequalities

$$A_1(1+|\eta|)^\gamma \leqslant |A(\eta)| \leqslant A_2(1+|\eta|)^\gamma, \quad A_1, A_2, \gamma > 0,$$

for all $\eta \in \mathbb{R}^p$; functions $\varphi_j(x)$, $j = 1,\dots,n$, are almost periodic with respect to x with given spectrum $\mathcal{M}$,

$$\mathcal{M} := \{\mu_k \in \mathbb{R}^p : d_1|k|^{\sigma_1} \leqslant |\mu_k| \leqslant d_2|k|^{\sigma_2}, k \in \mathbb{Z}^p, d_1, d_2, \sigma_1, \sigma_2 > 0\}.$$

The aim of our work is to establish conditions of the uniqueness and the existence of a solution to the problem (1), (2) depending on coefficients of factorization λ_j, $j = 1\dots,n$, in corresponding spaces of almost periodic with respect to x functions. The partial case of the problem (1), (2) was investigated in [1]. In general, such problems are ill-posed and their correct solvability is related to the problem of small denominators [2] to solve which we use the metric approach.

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THREE-DIMENSIONAL SIMULATIONS OF MAGNETIC FIELD IN SN 1572

The evolution of Supernova Remnants (SNRs) can be described by the system of magnetic hydrodynamics (MHD) equations. These equations convey the fact that mass, momentum, energy, and magnetic field are conserved in a physical system. Mathematically, MHD equations are a system of hyperbolic PDEs and generally cannot be solved in analytical form without significant simplifications that cannot fully account for most physical processes during the SNR evolution. In order to characterize the SNR evolution, one has to solve MHD equations numerically. We make use of PLUTO MHD code [1] that targets high Mach number flows with discontinuities in astrophysical fluid dynamics. Simulations in 3 dimensions even with a moderate spatial resolution (512x512x512) require significant computational resources and thus can only be performed on HPC (high-performance computing) clusters.

SN 1572 (Tycho) is one of few historical SNRs with the supernova explosion observed in detail. It made possible the identification of the supernova event as type Ia (thermonuclear explosion) and the discovery of the remnant itself. In order to simulate SNRs one needs to know details of the SN explosion (energy, mass, density distribution) and parameters of the circumstellar and interstellar medium: gas density, temperature, magnetic field, etc. The observations in radio and X-rays of SN 1572 indicate an inhomogeneous environment in the vicinity of the SNR. Also the observations give some hints on the structure of the environment which we can use to construct a 3D model of the SNR evolution. Having conducted the simulations we can then verify the model against the observed data, i.e. synthesize SNR maps in radio and X-rays from simulations data and compare these to the actual SNR observations.

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QUATERNION CORE INVERSE AND ITS GENERALIZATIONS

The core inverse of a complex matrix was introduced [1], recently.

A matrix $\mathbf{X} \in \mathbb{C}^{n\times n}$ is called *the core inverse* of $\mathbf{A} \in \mathbb{C}^{n\times n}$ if it satisfies $\mathbf{AX} = \mathbf{AA}^{\dagger}$, and $\mathcal{R}(\mathbf{X}) = \mathcal{R}(\mathbf{A})$, where $\mathbf{A}^{\dagger}$ means the Moore-Penrose inverse. Then, it is denoted by $\mathbf{A}^{\oplus}$.

Further, other generalizations of the core inverse, namely, core-EP inverse, DMP and CMP inverses, etc., were introduced and explored. In this talk, we extend their notions and give their representations for matrices over the quaternion skew field that have some features in comparison to complex matrices. Using row-column noncommutative determinants, we derive direct methods of their computing by their determinantal representations [2]. Determinantal representations of the core inverse and its generalizations for complex matrices were obtained separately in [3].

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THE STANDARD FORM OF MATRICES OVER IMAGINARY EUCLIDEAN QUADRATIC RINGS WITH RESPECT TO (Z,K)–EQUIVALENCE

Let $\mathbb{K} = \mathbb{Z}\left[\sqrt{k}\right]$ is a Euclidean quadratic ring and $\mathcal{E}(a) \in \mathbb{N}$ is the Euclidean norm of an element $a \in \mathbb{K}$ [1]. Let $M(n, \mathbb{K})$ be the ring of $(n \times n)$-matrices over the ring $\mathbb{K}$.

The matrices $A, B \in M\,(n, \mathbb{K})$ are called (z,k)-equivalent if exist such invertible matrices S over a ring of integers $\mathbb{Z}$ and Q over a quadratic ring $\mathbb{K}$, that $A = SBQ$.

This concept was introduced in [2] and it was established that a matrix A with $M(n, \mathbb{K})$ over the Euclidean quadratic rings (z,k)-equivalent transformations are reduced to lower triangular form: $T^A = TD^A$, where $D^A = \mathrm{diag}(\mu_1, \ldots, \mu_{\mathrm{n}})$ – the diagonal canonical form, T – the unitriangular lower matrix, namely $T = \left\|t_{ij}\right\|_1^n$, $t_{ii} = 1$, $t_{ij} = 0$ if $i < j$. The elements t_{ij} satisfy the conditions

1) $t_{ij} = 0$, if $\mu_i = 1$, $i, j = 1, \ldots, n$, $i > j$;

2) $\mathcal{E}(t_{ij}) = \frac{\mathcal{E}(\mu_i)}{\mathcal{E}(\mu_j)}$, if $t_{ij} \neq 0, i, j = 1, \ldots, n$, $i > j$.

This form T^A is called the standard form of the matrix A. The standard form defined non-uniqueness. There is problem selecting the classes of matrices for which the standard form is unique.

Theorem. *Let $\mathbb{K}$ be an imaginary Euclidean quadratic ring and a matrix $A \in M(n, \mathbb{K})$. If $\mathcal{E}(detA) < 4$ then the matrix A is (z,k)-equivalence to its canonical diagonal form D^A. This form is standard form for a matrix A with respect to (z,k)-equivalence and it is unique.*

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ESTIMATION OF FUNCTIONS BESOV SPACE BY EXTENDED LEGENDRE WAVELET AND ITS OPERATIONAL MATRIX OF INTEGRATION

In this paper, the function of Besov space, i.e., a more generalized space, is approximated by extended Legendre wavelet. The estimators obtained using the Euclidean norm as well as the Besov norm are very keen and sharper. A new operational matrix of integration is generated by using extended Legendre wavelet. Numerical examples are incorporated to show the accuracy and efficiency of the technique. The results obtained for different parameter values are compared with the Galerkin method. It is noted that the proposed numerical technique is quite logical. It demonstrates the utility and authenticity of the operational matrix of integration. The estimators and techniques of this paper generalize several known results of approximation of functions in wavelet analysis.

Theorem 1. *If $f \in B_q^{\alpha}(L_p), \alpha > 0, 1 \leq p, q < \infty$, and its extended Legendre wavelet expansion is $f(t) = \sum_{n=1}^{\mu^k} \sum_{m=0}^{\infty} \langle f, \psi_{n,m}^{(\mu)} \rangle \psi_{n,m}^{(\mu)}(t)$, then the wavelet approximations $E_{\mu^k,M}(f)$ of f by $(\mu^k, M)^{th}$ partial sum $(S_{\mu^k,M} f)(x)$ satisfies*

$$E_{\mu^k,M}(f) = \begin{cases} O\left(\frac{1}{\sqrt{k}\mu^{kM\alpha} M^{(1+\alpha)/q-1/2}}\right), & p > 2, \\ O\left(\frac{1}{\sqrt{k}\mu^{kM(\alpha+1/2-1/p)} M^{(1+\alpha)/q-1/2}}\right), & 1 \leq p < 2 \text{ and} \\ & \alpha > \frac{1}{p} - \frac{1}{2}. \end{cases}$$

Theorem 2. *If $f \in B_q^{\alpha}(L_p), 0 \leq \beta < \alpha, 1 \leq p, q < \infty$, then the approximation $E_{\mu^k,M}^{\beta}(f)$ of f by $(\mu^k, M)^{th}$ partial sum $S_{\mu^k,M}(f)$ of its extended Legendre wavelet expansion satisfies:*

$$E_{\mu^k,M}^{\beta}(f) = \begin{cases} O\left(\frac{1}{\mu^{kM(\alpha-\beta)} M^{(1+\alpha-\beta)/q-1/2}}\right), & p \geq 2, \\ O\left(\frac{1}{\mu^{kM(\alpha-\beta+1/p-1)} M^{(1+\alpha-\beta)/q-1}}\right), & 1 < p < 2 \text{ and} \\ & \alpha - \beta > 1 - \frac{1}{p}. \end{cases}$$

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PERTURBED MOTIONS OF A RIGID BODY UNDER THE ACTION OF UNSTEADY TORQUES

Perturbed motion of a rigid body, close to the Lagrange case, under the action of restoring and perturbation torques that are slowly varying in time is investigated in the work. The motion in Lagrange's case can be regarded as a generating motion of a rigid body, which takes into account the main torques acting on the body. The perturbed motions of a rigid body, close to the Lagrange case, were considered with the help of the averaging method in [1, 2]. We describe an averaging procedure for slow variables of a perturbed motion of a rigid body, close to the Lagrange case. Conditions for the possibility of averaging the equations of motion with respect to the nutation phase angle are presented and averaging procedure for slow variables of a perturbed motion of a rigid body in the first approximation is described. The averaged system of equations is obtained and qualitative analysis of motion is conducted.

As an example of the developed procedure, we investigate a perturbed motion, close to Lagrange's case, taking into account the torques acting on a rigid body from the external medium. The averaged system is integrated numerically for various initial conditions and parameters of the problem. The graphs of the solutions were built. A new class of rotational motions of a dynamically symmetric rigid body about a fixed point has been investigated with unsteady restoring and perturbation torques being taken into account.

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ON ONE PROBLEM OF THE INTERACTION OF MOVING OBJECTS

In the vacuum environment $\mathbb{R}^3$ is considered as a countable system of isolated and freely moving objects Z_j, each of which bears a certain potential p_j, while an arbitrary object of the system with P potential affects another object with p potential under the following law:

$$F = G\frac{P \cdot p}{1 + |r|^3} r^\circ,$$

where F is the force of interaction, G is the weight constant, r is a vector of the distance between objects; $r^\circ := r/|r|$, $|r|^2 := (r, r)$ is a square of the dot product in $\mathbb{R}^3$.

The determined objective is to study the force of interaction $F(t)$ at time t per unit of an object's potential Z_0 having been influenced by its immediate environment, provided that Z_0 is located at a origin of a coordinate system.

Since $F(t)$ is a variable with relatively fast and sharp deviations caused by an instantaneous change in local distribution of objects from the environment Z_0, it would be appropriate to consider $F(t)$ as a random variable.

It is proved that the probability distribution $W_t(F)$ for the force $F(t)$ has the structure

$$W_t(F) = \frac{a(t)}{\pi^2 \big(a(t)^2 + |F|^2\big)^2}, \quad F \in \mathbb{R}^3.$$

with certain a variable scale $a(t)$ and is a solution to the pseudo-differential equation

$$\partial_t u(x;t) + a'(t) \triangle_x^{1/2} u(x;t) = 0, \quad t \in (0;T],\, x \in \mathbb{R}^3,$$

where $\triangle_x$ is the Laplace operator acting on a variable x in a space $\mathbb{R}^3$.

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TIME-SPACE FRACTIONAL CAUCHY PROBLEM IN WEIGHT DISTRIBUTIONS

We study the Cauchy problem

$$u_t^{(\beta)} + (-\Delta)^{\gamma/2} u = g(t)F_0(x), \quad (x,t) \in \mathbb{R}^n \times (0,T] := Q, \tag{1}$$

$$\frac{\partial^{j-1}}{\partial t^{j-1}} u(x,0) = F_j(x), \;\; x \in \mathbb{R}^n, \;\; j = \overline{1,m} \tag{2}$$

with the Riemann-Liouville fractional derivative of order $\beta \in (m-1,m)$, $m,n \in \mathbb{N}$, $\beta < \gamma$ and $(-\Delta)^{\gamma/2}u$ determined by the use of the Fourier transform as follows $F[(-\Delta)^{\gamma/2}u] = |\lambda|^\gamma F[u]$.

Let $C^{k,(0)}(\bar{Q}) := \{v \in C^{[k]}(\bar{Q}) : (\frac{\partial}{\partial t})^j v(x,T) = 0, j = \overline{0,[k]}\}$ ($[k]$ be the integral part of k), $\mathcal{D}[0,T] = C^{\infty,(0)}[0,T]$, $\mathcal{D}(\overline{Q})$ be the space of functions from $C^{\infty,(0)}(\bar{Q})$ having compact supports, $\mathcal{E}'(\mathbb{R}^n) = [C^\infty(\mathbb{R}^n)]'$ be the space of distributions with compact supports.

Let $\varrho \in \mathcal{D}[0,T]$ be the nonnegative function, such that $\lim\limits_{t\to 0+} \varrho(t)/t = const$, $\varrho(x,t)$ ($(x,t) \in \bar{Q}$) be the nonnegative function from $\mathcal{D}(\bar{Q})$, such that $\lim\limits_{t\to 0+} \varrho(x,t)/t = \varrho_0(x)$. For $k \geq 0$ we define the following spaces $\mathcal{D}_k[0,T] = \{v \in C^{\infty,(0)}(0,T] : \varrho^{s-k} v^{(s)} \in C[0,T] \;\; \forall s \in \mathbb{Z}_+\}$, $\mathcal{D}_k(\bar{Q}) = \{v \in C^{\infty,(0)}(\bar{Q}) : \varrho^{s-k}(x,t) D^{(\alpha,s)} v \in C(\bar{Q}) \; \forall \alpha \in (\mathbb{Z}_+)^n, s \in \mathbb{Z}_+\}$.

Theorem 1. [1] *Assume that $F_j \in \mathcal{E}'(\mathbb{R}^n)$, $j = \overline{0,m}$, $k \geq 0$, $g \in \mathcal{D}'_{k+\beta}[0,T]$. Then there exists the unique solution $u \in \mathcal{D}'_k(\bar{Q})$ of the Cauchy problem (1), (2) with $m = 1$ and $m = 2$.*

The representation of the problem's solution by means of the Green vector-function is obtained.

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A CONDITION FOR GENERALIZED SOLUTIONS OF MATRIX PARABOLIC PROBLEMS TO BE CLASSICAL

In the finite multidimensional cylinder, we consider a linear differential parabolic problem for a Petrovskii $2b$-parabolic system, general boundary conditions, and homogeneous Cauchy data. All coefficients of the corresponding PDOs are complex-valued functions of class C^{∞}; the boundary of the base of the cylinder is also of class C^{∞}. In [1], we found a new exact sufficient condition under which generalized solutions to this problem are classical, i.e. they have the minimal regularity that allows us to calculate the left-hand sides of the problem via classical partial derivatives and via traces of continuous functions. The condition is formulated in terms of the belonging of the right-hand sides of the problem to some anisotropic Hörmander spaces. These spaces are parametrized with a pair of positive numbers s and $s/(2b)$ and a Borel measurable function $\varphi : [1,\infty) \to (0,\infty)$ such that φ and $1/\varphi$ are bounded on each compact subset of $[1,\infty)$ and that φ varies slowly at infinity in the sense of Karamata, i.e. $\varphi(\lambda r)/\varphi(r) \to 1$, as $r \to \infty$, whenever $\lambda > 0$. The $\varphi(\cdot) \equiv 1$ case yields anisotropic Sobolev spaces. The use of the function parameter φ allows us to achieve the minimal admissible values of the number parameters in the condition found, which is not possible within the framework of (anisotropic) Sobolev spaces or Hölder spaces. For these values, the condition

$$\int_{1}^{\infty} \frac{dr}{r\varphi^{2}(r)} < \infty$$

implies that the solutions are classical. This condition is sharp.

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FUNCTIONAL CALCULUS IN ALGEBRA OF ω–ULTRADISTRIBUTIONS

A fuctional calculus is a theory that studies how to construct functions depending on operators. There are many different approaches to construct a functional calculus for one operator action on a Banach space. The Hille-Phillips fuctional calculus for functions of several variables was developed in [1]. We develope Hille-Phillips type functional calculus for generators of strongly continuous groups. As a symbol classes of such calculus we use the algebra of ω–ultradistributions $\mathcal{E}'_*$ [2]. We prove a differential property of constructed calculus and discribe its image with the help of the commutant of shift group.

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CONTINUITY OF WAVELET TRANSFORM ON SOBOLEV TYPE SPACES

This presentation is concerned with the study of the continuity of wavelet transform involving fractional Hankel transform on certain Zemanian type spaces. The mapping properties of the fractional wavelet transform is also discussed on Sobolev type space. Particular cases are also considered.

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ON SINGLE LAYER POTENTIALS FOR SOME PSEUDO-DIFFERENTIAL EQUATIONS

Let $\mathbf{A}$ be the pseudo-differential operator defined by its symbol $-(Q\xi,\xi)^{\frac{\alpha}{2}}$, $\xi \in \mathbb{R}^d$, where $Q = PP^T$ with some invertible real $d\times d$-matrix, $\alpha \in (1,2)$ is some constant. The function $g(t,x,y) = \frac{1}{(2\pi)^d}\int_{\mathbb{R}^d} e^{i(\xi,x-y)-t(Q\xi,\xi)^{\frac{\alpha}{2}}}d\xi$ is the transition probability density of the Markov process $x(t) = Px_0(t)$, where $(x_0(t))_{t\geq 0}$ is a rotationally invariant α-stable stochastic process. Let S be some surface in $\mathbb{R}^d$ and $\psi : (0;+\infty)\times S \to \mathbb{R}$ be some function. The function $v(t,x) = \int_0^t d\tau \int_S g(t-\tau,x,y)\psi(\tau,y)d\sigma_y$ is called *the single layer potential* for the pseudo-differential equation $\frac{\partial u(t,x)}{\partial t} = \mathbf{A}u(t,\cdot)(x)$.

Theorem 1. *Let a surface S in $\mathbb{R}^d$ belong to the class $H^{1+\gamma}$ with some fixed $\gamma \in (0;1)$ and satisfy one of the conditions: it is bounded and closed or it is unbounded, but could be presented by an explicit equation in some coordinate system. Let a continuous function $(\psi(t,x))_{t\geq 0, x\in S}$ satisfy the inequality $|\psi(t,x)| \leq K_T t^{-\beta}$ on each set of the kind $(0;T]\times S$, $T > 0$ with some constants $\beta < 1$ and $K_T > 0$. Therefore, $(v(t,x))_{t\geq 0, x\in\mathbb{R}^d}$ is correctly defined; it is the continuous function and satisfies the equation $\frac{\partial v(t,x)}{\partial t} = \mathbf{A}v(t,\cdot)(x)$ for all $(t;x)$ from the domain $(0;\infty)\times(\mathbb{R}^d\setminus S)$.*

Let us introduce the operator $\mathbf{B}$ by its symbol $i|\xi|^{\alpha-2}\xi$, $\xi \in \mathbb{R}^d$ and denote the operator $(\nu,\mathbf{B})$, where $\nu \in \mathbb{R}^d$ is a unique vector, by $\mathbf{B}_\nu$.

Theorem 2. *Let the conditions of Theorem 1 be fulfilled. Then for all $t \geq 0$, $x \in S$ the following equality ($\nu(x) = Qn_x$, $\hat{\nu}(x) = Q^{-1}n_x$, $n_x \in \mathbb{R}^d$ is a normal vector to one side of the surface S at the point $x \in S$)*

$$\lim_{z\to x\pm}\mathbf{B}_{\hat{\nu}(x)}v(t,\cdot)(z) = \mp\frac{1}{2}\psi(t,x)+\int_0^t d\tau\int_S \mathbf{B}_{\hat{\nu}(x)}g(t-\tau,\cdot,y)(x)\psi(\tau,y)d\sigma_y,$$

holds true, where $z \to x\pm$ means $z = x+\delta\nu(x)$ and $\delta \to 0\pm$.

Details of these theorems proofs will be able to find in article [1].

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ELASTIC-PLASTIC PROBLEM OF STATE ROCK MASS NEAR HYDRAULIC CRACK

This article is devoted to the study of stresses near a fracture. The hydraulic fracturing technology consists in creating a highly conductive fracture in the target formation under the action of a fluid supplied to it under pressure to ensure the flow of produced gas to the bottom of the well [1].

The initial stress field is due to the dead weight of the rocks. The first stress field is determined from the conditions that the crack crosses the coal seam. Stresses, displacements, and strains are determined from Lechnitsky's representations in the form of complex functions [2, 3]. The solution to the problem is found using the Signorini–Keldysh–Sedov integral formula.

The Second stress field is determined for the fracture, which passes through the reservoir and rocks. The complex potentials of Lehnitsky are rewritten for a turn [2, 3]. The problem reduces to a mixed one for the left and right half-planes, and the boundary conditions are written out on the y axis. The potentials are determined by the Signorini–Keldysh–Sedov formula [2, 4].

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SUPERCYCLIC OPERATORS ON FREE BANACH SPACE

Let X be a complex topological vector space with $dim(X) > 1$ and $L(X)$ the space of all continuous linear operators on X. If $T \in L(X)$, then the orbit of a vector $x \in X$ under T is the set $Orb(T, x) := T^n x : n \geq 0$. The operator T is said to be supercyclic if there is some vector $x \in X$ such that $COrb(T, x) = \alpha T^n x : \alpha \in C, n \geq 0$ is dense in X. Such vector x is called a supercyclic vector for T , and the set of all supercyclic vectors for T is denoted by $SC(T)$. We say that an operator $T \in L(X)$ satisfies the Supercyclicity Criterion if there exist an increasing sequence of integers (n_k), a sequence (a_{n_k}) of nonzero complex numbers, two dense sets $X_0, Y_0 \in X$ and a sequence of maps $S_{n_k} : Y_0 \to X$ such that:

1) $\alpha_{n_k} T^{n_k} x$ for any $x \in X_0$;
2) $\alpha_{n_k}^{-1} S_{n_k} y \to 0$ for any $y \to Y_0$;
3) $T^{n_k} S_{n_k} y \to y$ for any $y \to Y_0$.

Let X and Y be metric spaces. A map $f : X \to Y$ is Lipschitz if there exists a constant $L_f \geq 0$ such that

$$\rho(f(p), f(q)) \leq L_f \rho(p, q)$$

for all $p, q \in X$. We denote by $Lip_0(X, Y)$ the space of all Lipschitz maps between metric spaces X with fixed point θ_x and metric space Y, with fixed point θ_y. It is known that for an arbitrary metric pointed space (X, θ_x) there is a unique (up to isometrical isomorphism) Banach space $B(X)$ and a Lipschitz embedding $\nu : X \to B(X)$ such that for every normed space E and any map $f(x) \in Lip_0(X, E)$ there is a linear operator $\widetilde{f}(x) : B(X) \to E$ with $\widetilde{f}(\nu(x)) = f(x), x \in X$.

Theorem. Let X be a separable Frechet space. If $T : X \to X$ is a supercyclic operator satisfying the Supercyclicity Criterion, then $\widehat{T} : B(X) \to B(X)$ is also a supercyclic operator and satisfies the Supercyclicity Criterion.

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ON ONE APPROACH OF BOUNDARY VALUE PROBLEM INVESTIGATION FOR EQUATIONS OF THE HYPERBOLIC TYPE IN DOMAINS WITH A COMPLEX STRUCTURE OF THE BOUNDARY AND A PRE-HISTORY

Let on the space xOy we observe the domain $D = D_1 \cup D_2$, where $D_1 = \{(x,y)\,|x \in (\kappa_1(y), x_2]\,, y \in (y_1, y_2]\}\,$, $D_2 = \{(x,y)\,|x \in (x_1, x_2]\,,$ $y \in (g_2(x), y_1]\}$, where $y = g_i(x)(x = \kappa_i(y))$, $i = 1,2$ are 'free' curves, moreover $g_2^{'}(x) > 0, x \in [x_1, x_2]$, $g_1^{'}(x) < 0, x \in [x_0, x_1]$, $g_2(x_i) = y_{i-1}$, $g_1(x_j) = y_{2-j}, j = 0,1$. **Problem settings** [1]: in the space of functions $C^*(\overline{D}_1) := C^{(1.1)}(D_1) \cap C(\overline{D}_1)$ to find a solution of the equation $L^{(1.1)}_{(a_{i,1})} U_1(x,y) := U_{1_{xy}}(x,y) + a_{1,1}(x,y)U_{1_x}(x,y) + a_{2,1}(x,y)U_{1_y}(x,y) = f_1(x,y,U_1(x,y)) := f_1\left[U_1\left(x,y\right)\right]$, which satisfies the conditions $U_1(x, g_1(x)) = \psi_2(x), U_1(x,y_2) = \varphi(x)$, $x \in [x_0, x_1]$, $U_1(x,y_1) = U_2(x,y_1)$, $x \in [x_1, x_2]$, where $U_2(x,y) \in C^*(\overline{D}_2)$ is the solution the Darboux problem $L^{(1.1)}_{(a_{i,2})} U_2(x,y) := f_2(x,y,U_2(x,y)) := f_2\left[U_2\left(x,y\right)\right], U_2(x_1,y) = \mu(y), y \in [g_2(x), y], U_2(x, g_2(x)) = \psi_1(x)$, $x \in [x_1, x_2]$, and appropriate condition of consistency are true $\psi_2(x_0) = \varphi(x_0), \psi_2(x_1) = \mu(y_1), \mu(y_0) = \psi_1(x_1)$. Under the condition, that $a_{1,i}(x,y) \in C^{(1.0)}(D_i)$, $a_{2,i}(x,y) \in C^{(0.1)}(D_i)$, $i = 1,2$, $a_{1,i_x}(x,y) = a_{2,i_y}(x,y)$, and functions $F_i\left[U_i(x,y)\right] \in C_1(\overline{B}_i)$, $F_i : \overline{B}_i \to \mathbb{R}$, $\overline{B}_i \subset \mathbb{R}^3$, $F_i\left[U_i\left(x,y\right)\right] := f_i\left[U_i\left(x,y\right)\right] + [a_{1,i_x}(x,y) + a_{1,i}(x,y)\, a_{2,i}(x,y)]\, U_i\left(x,y\right)$, (definition of space $C_1(\overline{B}_i)$ is given in [1, p.206]), it has been possible to:

1. build a constructive rapidly-convergent iteration method investigation and approximate solution of problem,
2. receive sufficient conditions of existence, uniqueness and constancy of sign solution under consideration problem and proof theorem of comparison

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ON CONTINUITY IN A PARAMETER OF SOLUTIONS TO BOUNDARY-VALUE PROBLEMS IN SLOBODETSKY SPACES

Let numbers $p \in (1, \infty), m \in \mathbb{N}, r \in \mathbb{N}, s \in \mathbb{R}_+ \setminus \mathbb{Z}_+, s := [s] + \{s\}$, where $[s] \in \mathbb{Z}_+$ and $\{s\} \in (0, 1)$. We investigate a parameter-dependent linear boundary-value problem of the form

$$L(\varepsilon)y(t,\varepsilon) \equiv y^{(r)}(t,\varepsilon) + \sum_{j=1}^{r} A_{r-j}(t,\varepsilon)y^{(r-j)}(t,\varepsilon) = f(t,\varepsilon),\ a \leq t \leq b,$$

$$B(\varepsilon)y(\cdot,\varepsilon) = c(\varepsilon).$$

For each number $\varepsilon \in [0, \varepsilon_0)$, $y(\cdot,\varepsilon) \in (W_p^{s+r})^m$ is an unknown vector-valued function, whereas the matrix-valued functions $A_{r-j}(\cdot,\varepsilon) \in$ $\in (W_p^s)^{m\times m}$, where $j \in \{1, \dots, r\}$, vector-valued function $f(\cdot,\varepsilon) \in (W_p^s)^m$, continuous linear operator $B(\varepsilon) : (W_p^{s+r})^m \to \mathbb{C}^{rm}$ and vector $c(\varepsilon) \in \mathbb{C}^{rm}$ are arbitrarily given.

In paper [1], we proved these problems are Fredholm, and obtained conditions that are sufficient for their well-posedness and continuity in the parameter of their solutions in Slobodetsky spaces. Now, we obtain a constructive criterion under which the solutions to these problems are continuous with respect to the parameter in the normed space $(W_p^{s+r})^m$. A two-sided estimate for the degree of convergence of these solutions was obtained there. We also introduce and investigate a new broad class of multipoint linear boundary-value problems for systems of ordinary $r \geq 2$ order differential equations with solutions in the Slobodetsky space. For such problems we establish sufficient conditions for their solutions to be continuous in the parameter at $\varepsilon = 0$ in the space $(W_p^{s+r})^m$.

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FROM THE N-POINT GEOMETRY UP TO THE INVERSE PROBLEM IN THE CALCULUS OF VARIATIONS

It is shown that a quite natural idea about the n-point geometry of Skorobohat'ko immediately evokes the necessity of the inverse variational problem machinery application in the realm of the higher order Ostrohrads'kyj mechanics. Crucial aspects of the corresponding tools are recalled. The application of the inverse variational calculus to Euclidian spaces is proved to yearn the implementation of symmetry consideration alongside with them. The definition of the symmetry of a differential equations is given in terms of Phaff exterior differential systems theory. The difference between the existence of a locally variational equation of motion and the existence of a global Lagrange function for such equation is emphasized. Next problem concerns to the possibility of generalizing some variational equations of motion obtained in Euclidian space to covariant variational equations in Riemannian space. Some preliminary assertions on this problem are covered. Certain examples are produced.

Theorem. *In 2-dimensional Riemannian space the variational problem with the action integral* $\int(k - m\|\boldsymbol{u}\|)d\tau$ *produces the set of geodesic circles and generalizes the only possible invariant variational equation in the Euclidian plane with the Frenet curvature* k *as its first integral*

Theorem. *In the Euclidian space of dimension greater than 2 there does not exist an invariant Lagrange function to produce a third order variational equation.*

Theorem. *In three dimensional Euclidian space there exists the only third order locally variational invariant differential equation*

$$-\frac{\ddot{\boldsymbol{u}}\times\boldsymbol{u}}{\|\boldsymbol{u}\|^3}+3\,\frac{\dot{\boldsymbol{u}}\times\boldsymbol{u}}{\|\boldsymbol{u}\|^5}\,(\dot{\boldsymbol{u}}\cdot\boldsymbol{u})-\frac{\mu}{\|\boldsymbol{u}\|^3}\,\big[(\boldsymbol{u}\cdot\boldsymbol{u})\,\dot{\boldsymbol{u}}-(\dot{\boldsymbol{u}}\cdot\boldsymbol{u})\,\boldsymbol{u}\big]=\boldsymbol{0}\,.$$

The integral curves are helices with the torsion $\kappa = -\mu$*.*

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EFFECTIVE DYNAMIC CONDITIONS FOR AN INTERFACE NON-CONTRAST INCLUSION BETWEEN TWO ELASTIC MATERIALS

The influence of partial imperfectly bonded interfaces between two bodies on the scattering amplitude of SH-waves is investigated. For this purpose, a linear elastic body consisting of two half-spaces and a thin inclusion at an interface is considered. The inclusion is assumed to be non-contrast in comparison with the materials of half-spaces. Introducing a small parameter that characterizes the relative of the thickness inclusion and using matched asymptotic expansions [1-3], asymptotically accurate models of the dynamic interaction of a thin inclusion with adjacent components of the composition are proposed. At the order zero of the asymptotic expansions, we obtain a perfect interface model, which prescribes the vanishing of the jumps in the displacement and their normal derivatives. At a higher order (the first term in the expansions), an imperfect interface model we obtain, with an effective dynamic contact condition involving displacement and their derivatives at order zero. The amplitude of scattered waves is obtained by using the Green's integral formula.

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SUFFICIENT CONDITIONS FOR THE EMERGENCE OF SOLUTIONS OF PERTURBED BOUNDARY VALUE PROBLEMS FOR QUASI-DIFFERENTIAL EQUATIONS WITH MEASURES AS COEFFICIENTS

We study the solvability conditions of a perturbed linear nonhomogeneous boundary value problem (BVP) $\mathcal{P}_\varepsilon$:

$$Ly(x) = f(x) + \varepsilon\sigma(x)y(x), \tag{1}$$

$$l_k y(\cdot) = \eta_k + \varepsilon\tilde{l}_k y(\cdot),\ k = \overline{1,m}. \tag{2}$$

Here $Ly(x) := \sum_{i=0}^{p}\sum_{j=0}^{q}(-1)^{q-j}\left(a_{ij}(x)y^{(p-i)}\right)^{(q-j)}$ is a quasi-differential expression of order $n\,(= p+q)$, l_k and $\tilde{l}_k$ are linear functionals defined in the space $BV^+[a,b]$ of right-continuous functions of bounded variation on $[a,b]$, $\eta_k \in \mathbb{R}$, $\varepsilon \geqslant 0$ is a small parameter. We suppose that the following conditions hold: (A) a_{00}^{-1} is bounded and measurable on $[a,b]$ function; (B) a_{i0}, a_{0j} are Lebesque integrable square on $[a,b]$ functions $(i = \overline{1,p},\ j = \overline{1,q})$; (C) a_{ij} $(i = \overline{1,p},\ j = \overline{1,q})$, σ, and f are measures on $[a,b]$, i. e. $a_{ij} = b'_{ij}$, $\sigma = \omega'$, and $f = g'$, where $b_{ij}, \omega, g \in BV^+[a,b]$, so the differentiation and the equality in (1) are understood in the generalized sense.

We have the critical case [1] when the generating (unperturbed) BVP $\mathcal{P}_0$ is unsolvable for some $g \in BV^+[a,b]$ and $\eta_k \in \mathbb{R}$ $(k=\overline{1,m})$. It is of interest to analyse whether it is possible to make this problem solvable by means of linear perturbations and if this is possible, then of what kind should the perturbations $\omega(x)$ and $\tilde{l}_k$ $(k=\overline{1,m})$ be. In this critical case, we obtain a constructive condition for the emergence of solutions of $\mathcal{P}_\varepsilon$ and construct an iterative procedure for finding these solutions in the form of Laurent series in powers of a small parameter ε containing one term with a negative power of ε.

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FUNDAMENTAL SOLUTIONS OF ULTRAPARABOLIC KOLMOGOROV-TYPE EQUATIONS WITH THREE GROUPS OF SPATIAL VARIABLES AND DEGENERATION ON THE INITIAL HYPERPLANE

The numbers n, n_1, n_2, n_3 are given natural such that $n_1 \geq n_2 \geq n_3 \geq 1$ and $n = n_1+n_2+n_3$. The spatial variable $x := (x_1, x_2, x_3) \in \mathbb{R}^n$ consist of three groups of variables $x_j := (x_{j1}, \ldots, x_{jn_j}) \in \mathbb{R}^{n_j}$, $j \in \{1, 2, 3\}$.

Consider the equation

$$(\alpha(t)\partial_t - \beta(t)(\sum_{j=1}^{n_2} x_{1j}\partial_{x_{2j}} + \sum_{j=1}^{n_3} x_{2j}\partial_{x_{3j}} + \sum_{j,l=1}^{n_1} a_{jl}(t,x)\partial_{x_{1j}}\partial_{x_{1l}} +$$

$$+\sum_{j=1}^{n_1} a_j(t,x)\partial_{x_{1j}}) - a_0(t,x))u(t,x) = 0,\ (t,x) \in (0,T] \times \mathbb{R}^n.$$

Assume that coefficients $a_{jl}, a_j, a_0 : [0,T] \times \mathbb{R}^n \to \mathbb{C}$ satisfy the conditions from paper [1]. The functions α and β are continuous on $[0,T]$ which satisfy the conditions: $\alpha(t) > 0$, $\beta(t) > 0$ for each $t \in (0,T]$, $\alpha(0)\beta(0) = 0$, β is monotonically nondecreasing function. Under these assumptions the fundamental solution for the ultraparabolic Kolmogorov equation with degeneration on the initial hyperplane is constructed and investigated by using of modification of the classical Levi method [2].

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ASYMPTOTIC APPROXIMATIONS FOR EIGENVALUES AND EIGENFUNCTIONS OF A SPECTRAL PROBLEM IN A THIN GRAPH-LIKE JUNCTION WITH A CONCENTRATED MASS IN THE NODE

A spectral problem is considered in a thin $3D$ graph-like junction that consists of three thin curvilinear cylinders that are joined through a domain (node) of the diameter $\mathcal{O}(\varepsilon)$, where ε is a small parameter. A concentrated mass with the density $\varepsilon^{-\alpha}$ $(\alpha \geq 0)$ is located in the node. The asymptotic behaviour of the eigenvalues and eigenfunctions is studied as $\varepsilon \to 0$, i.e. when the thin junction is shrunk into a graph.

There are five qualitatively different cases in the asymptotic behaviour $(\varepsilon \to 0)$ of the eigenelements depending on the value of the parameter α. In the talk three cases will be reported, namely, $\alpha = 0$, $\alpha \in (0, 1)$, and $\alpha = 1$.

Using multiscale analysis, asymptotic approximations for eigenvalues and eigenfunctions are constructed and justified with a predetermined accuracy with respect to the degree of ε. For irrational $\alpha \in (0, 1)$, a new kind of asymptotic expansions is introduced.

These approximations show how to account the influence of local geometric inhomogeneity of the node and the concentrated mass in the corresponding limit spectral problems on the graph for different values of the parameter α.

The results are published in [1].

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MATHEMATICAL MODELING OF THE PROBLEM OF DETERMINING TEMPERATURE IN A LAYERED OBJECT IN CONTACT FOR FUNCTIONALLY GRADIENT MATERIALS

There is a problem of developing mathematical models for describing the thermomechanical behavior of structural elements of mechanical and tribological systems made of functionally gradient and thermosensitive materials with physical and mechanical characteristics dependent on temperature, under the combined action of local force and temperature loads. There are also problems in creating algorithms for calculating the stress-strain state of such bodies and in determining the tribological properties of the functional surfaces of promising materials and coatings in the conditions of mechanical contact when the counterbody is slides. There are significant achievements in the field of technology for creating new properties, for example, from functionally gradient materials, however the developed mathematical models for solving boundary value problems require computer implementation. We solve a boundary value problem for a layered medium that determines the temperature depending on the boundary conditions (taking into account the movement of the indenterd). We study the change in the temperature field in the coating of functionally gradient materials in the case of indenter movement on the surface of the coating. We use the theoretical approaches from [1]–[3]. We create a calculation algorithm and build graphical dependences of the temperature change in the coating for various temperature parameters.

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NON-CLASSICAL TRANSFORMATION OPERATORS FOR STURM-LIOUVILLE OPERATORS

In the Hilbert space $L_2(\mathbb{R})$, we consider the self-adjoint Schrödinger operator T generated by the differential expression

$$\mathfrak{t}(y) = -\left(\frac{1}{p}\frac{d}{dx}p^2\frac{d}{dx}\frac{1}{p}\right)y$$

with the function

$$p(x) = \exp\left\{\sum_{j:\xi_j>x}\alpha_j\right\},$$

where $(\alpha_j)_{j=1}^n, (\xi_j)_{j=1}^n$– sequence of real numbers, and $(\alpha_j)_{j=1}^n$– belongs to $\ell_1(\mathbb{Z})$.

In this talk we present a construction of non-classical transformation operators for the operator T.

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ANALYTICAL AND *-ANALYTICAL APPROXIMATION IN NORMED SPACES

The main result from approximation continuous function by analytic in real separable Banach space was obtained by Kurzweil in paper [1] In paper [2] obtain a counterpart of Kurzweil's Theorem for a complex Banach space. In paper [3] with Ravsky A. obtaned next generalizations. Let X be a real separable normed space X admitting a separating polynomial. Each continuous function from a subset A of X to a real Banach space can be uniformly approximated by restrictions to A of functions which are analytic on open subsets of X. Each continuous function to a complex Banach space from a complex separable normed space admitting a separating $*$-polynomial can be uniformly approximated by $*$-analytic functions.

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APPROXIMATION BY PARAMETRIC EXTENSION OF PHILLIPS-OPERATORS INVOLVING THE APPELL POLYNOMIALS

In the present work, we study the approximation properties of Phillips operators of Dunkl analogue by including the Appell polynomials. For this purpose, first we introduced the Jakimovski-Levitian type new Phillips operators by involving the new parameter $\lambda \in [0, \infty)$ then discuss the convergence properties in weighted space, Lipschitz spaces and K-functional approximation by use the modulus of continuity. Furthermore, we have also discuss the Voronovskaja type approximation results.

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LOCAL TWO-POINT PROBLEM FOR THE EQUATION WITH THE OPERATOR OF GENERALIZED DIFFERENTIATION

Let $\{m_n\}_{n=1}^{\infty} \subset \mathbb{C}\backslash\{0\}$ be a sequence such that $\lim\limits_{n\to\infty} \frac{|m_n|}{n} = b$ for some $b > 0$. Function $F(z) = 1 + \sum\limits_{n=1}^{\infty} \frac{z^n}{m_1 \ldots m_n}$ is the entire function of the exponential type $1/b$ and generates [1] the operation Gel'fond-Leont'ev generalized differentiation D_F. For power series $f(z) = \sum\limits_{n=0}^{\infty} a_n z^n$ the action $D_F[f(z)]$ defined by equality $D_F[f(z)] = \sum\limits_{n=0}^{\infty} m_{n+1} a_{n+1} z^n$.

Let H be a separable Hilbert space over $\mathbb{C}$ with Hermitian scalar product $\langle\cdot,\cdot\rangle_H$ and base $\{e_k\}_{k=1}^{\infty}$ be an orthonormal system respect to this product. For the sequence $\{\lambda_k\}_{k=1}^{\infty} \subset \mathbb{C}$, such that $\lim\limits_{k\to\infty} |\lambda_k| = \infty$, we denote the linear operator $A : \mathcal{D}om(A) \to H$, $\mathcal{D}om(A) \subset H$, by the rule $Ae_k = \lambda_k e_k$, $k \in \mathbb{N}$.

Let consider such local two-point problem

$$\mathcal{D}_F^2 u(z) = Au(z), \quad z \in B = \{z \in \mathbb{C} : |z| < 1\}, \tag{1}$$

$$u(0) = \varphi_1, \quad u(T) = \varphi_2, \quad |T| \le 1, \tag{2}$$

where $\varphi_1, \varphi_2 \in H$, $u : \overline{B} \to H$ is unknown continuous function such that $u : B \to H$ is analytic and $\mathcal{D}_F^2 u(z) = \sum\limits_{k=1}^{\infty} D_F^2 [\langle u(z), e_k\rangle_H] e_k$. We obtained the conditions for solvability of the problem (1), (2) if the sequences $\{m_n\}_{n=1}^{\infty}$, $\{\lambda_k\}_{k=1}^{\infty}$ belong to special classes.

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STABILITY OF THE COSMOLOGICAL PERTURBATIONS OF INTERACTING DARK COMPONENTS IN THE RADIATION DOMINATED EPOCH

The evolution of cosmological perturbations is studied in the model of Universe where dark energy non-gravitationally interacts with dark matter (DE-DM interaction). Such modification of cosmological model causes the instabilities of the non-adiabatic mode of cosmological perturbations of dark components which occur in the radiation dominated epoch when the Hubble horizon is small in comparison to the perturbation scale. This instabilities are present in the models of dark energy with constant equation of state parameter and with DE-DM coupling proportional to the density of dark matter. So we used the model of dynamical dark energy parameters of which can be tuned in such way that the non-adiabatic perturbations are stable. Using the Linard-Chipart criterion the constraints on DE-DM coupling strength and other parameters were obtained. These constraints allowed to solve the cosmological perturbation equations numerically and to study the modification of perturbations' evolution caused by DE-DM interaction.

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ON THE NONEXISTENCE OF SOLUTION OF THE DEGENERATE TWO-POINT IN TIME PROBLEM FOR NONHOMOGENEOUS PDE

Two-point problem for nonhomogeneous PDE

$$\frac{\partial^2 U}{\partial t^2} + 2\,a\Big(\frac{\partial}{\partial x}\Big)\frac{\partial U}{\partial t} + b\Big(\frac{\partial}{\partial x}\Big)U = f(t,x), \tag{1}$$

with local homogeneous two-point conditions

$$\begin{aligned} A_1\Big(\frac{\partial}{\partial x}\Big)U(0,x) + A_2\Big(\frac{\partial}{\partial x}\Big)\frac{\partial U}{\partial t}(0,x) = 0, \\ B_1\Big(\frac{\partial}{\partial x}\Big)U(h,x) + B_2\Big(\frac{\partial}{\partial x}\Big)\frac{\partial U}{\partial t}(h,x) = 0 \end{aligned} \tag{2}$$

in domain $(t,x) \in \mathbb{R}^{1+s}$, $s \in \mathbb{N}$, is investigated by differential-symbol method [1].

In equation (1), the differential expressions $a\big(\frac{\partial}{\partial x}\big)$ and $b\big(\frac{\partial}{\partial x}\big)$ are such that the corresponding expressions $a(\nu)$ and $b(\nu)$ are entire functions, $f(t,x)$ is given nonzero function. The coefficients $A_i\left(\frac{\partial}{\partial x}\right)$ and $B_i\left(\frac{\partial}{\partial x}\right)$, $i \in \{1,2\}$ in conditions (2) are differential polynomials with complex coefficients, moreover their symbols $A_i(\nu)$ and $B_i(\nu)$ for each $\nu \in \mathbb{C}^s$ satisfy inequalities:

$$\sum_{i=1}^{2} |A_i(\nu)|^2 \neq 0, \quad \sum_{i=1}^{2} |B_i(\nu)|^2 \neq 0.$$

In the class of entire functions the nonexistence conditions of the solution of problem (1), (2) are found. The case when the characteristic determinant of the problem identically equals zero is considered.

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ON LINEAR CONVEXITY IN COMMUTATIVE ALGEBRAS

We consider a commutative and associative algebra $\mathcal{A}$ with unit over the field of real numbers. Let $\dim\mathcal{A} = n$ and let elements $\{e_k\}_{\underline{k=1}}^{n}$ be a basis of $\mathcal{A}$ such that there exist the inverse elements e_k^{-1}, $k = \overline{1,n}$. Let $\Gamma^p = (\gamma_{lk}^p)$, $p = \overline{1,n}$, be matrixes of structure constants of $\mathcal{A}$ with respect to the basis, i. e. $e_k e_l = \sum\limits_{p=1}^{n} \gamma_{lk}^p e^p$, $k, l = \overline{1,n}$. We suppose that among matrixes Γ^p there is at least one that is not degenerate.

The notion of linearly convex domains in multi-dimensional complex space $\mathbb{C}^n$ (1) is generalized on the multi-dimensional Euclidean space $\mathcal{A}^m$ that is the Cartesian product of algebras $\mathcal{A}$. In terms of nonnegativity and positivity of formal quadratic differential forms in $\mathcal{A}$, the necessary and sufficient conditions of the local $\mathcal{A}$-linear convexity of domains with smooth boundary in $\mathcal{A}^m$ are obtained respectively. These conditions are the generalization of the analytical conditions of local linear convexity of domains with smooth boundary in $\mathbb{C}^n$ (2). Similar conditions where also obtained for non-commutative Clifford algebras and generalized quaternions algebras (3).

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INTEGRAL MANIFOLDS AND DECOMPOSITION OF SYSTEMS OF MULTISCALE LINEAR SINGULARLY PERTURBED EQUATIONS

We investigate a system of linear singularly perturbed differential equations with many small parameters

$$\prod_{j=0}^{i} \varepsilon_j \dot{x}_i = \sum_{j=0}^{k} A_{ij} x_j,\ i = \overline{0,k},$$

where $x_i \in \mathbb{R}^{n_i},\ i = \overline{0,k},\ \ A_{ij} = A_{ij}(t),\ i,j = \overline{0,k}$, are matrices $n_i \times n_j$, $\varepsilon_0 = 1,\ \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k$ are small positive parameters.

The decomposition scheme algorithm contains k steps of successive splitting based on the ideas of the integral manifold method [1-2]. A method for constructing a sequence of substitutions has been developed, that take the original system to a set of independent subsystems [3]

$$\begin{cases} \dot{y}_0^k = B_{00}^k y_0^k, \\ \prod_{j=0}^{i} \varepsilon_j \dot{y}_i^{k+1-i} = B_{ii}^{k+1-i} y_i^{k+1-i}, i = \overline{1,k}. \end{cases}$$

Since the exact finding of the splitting transformation is possible only in the simplest cases, the paper proposes and substantiates the possibility of efficient construction of asymptotic expansions of transformation coefficients by powers of small parameters.

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REPRESENTATION OF A ONE CLASS FUNCTIONS OF TWO VARIABLES BY BICONTINUED FRACTIONS

Let a function $u(z,w) = f(z)h(w)$ be defined on the compact set $\mathbf{K} \subset \mathbb{C}^2$. We got a representation of a function u as the product of two Thiele continued fractions in the neighbourhood of a point $(z_*, w_*) \in \mathbf{K}$

$$\mathbf{B}(z,w;fh) = \left(b_0(z_*;f) + \mathop{\mathbf{K}}_{n=1}^{\infty} \frac{z - z_*}{b_n(z_*;f)}\right)\left(b_0(w_*;h) + \mathop{\mathbf{K}}_{n=1}^{\infty} \frac{w - w_*}{b_n(w_*;h)}\right).$$

We call this product a Thiele bicontinued fraction. Similarly, the function u can be represented by a bicontinued C–fraction

$$\mathbf{C}(z,w;fh) = \left(a_0(z_*;f) + \mathop{\mathbf{K}}_{n=1}^{\infty} \frac{a_n(z_*;f)(z - z_*)}{1}\right)\times$$

$$\times\left(a_0(w_*;h) + \mathop{\mathbf{K}}_{n=1}^{\infty} \frac{a_n(w_*;h)(w - w_*)}{1}\right).$$

Some new properties of Thiele reciprocal derivatives, Thiele continued fractions and regular C–fractions are proved. The possibility of representation of functions of this class by bicontinued fractions is shown. The representations of functions $u_1(z,w) = (\delta + \beta z)^{\alpha}\,\mathrm{tg}(\varepsilon + \gamma w)$, where $\alpha \in \mathbb{C}\backslash\{\mathbb{Z}\}$, $\beta, \gamma, \delta, \varepsilon \in \mathbb{C}\backslash\{0\}$ and $u_2(z,w) = e^{\alpha z}\ln(\beta + \gamma w)$, where $\alpha, \beta, \gamma \in \mathbb{C}\backslash\{0\}$ are obtained as examples. Domains of convergence and uniform convergence of obtained bicontinued fractions to the function are indicated.

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A SINGULAR NONLINEAR BOUNDARY VALUE PROBLEM ON REAL LINE WITH ASYMPTOTIC CONDITIONS AT INFINITY

Let $A(\cdot) \in \mathrm{C}(\mathbb{R} \mapsto \mathrm{Hom}(\mathbb{R}^n))$, $f(\cdot,\cdot) \in \mathrm{C}(\mathbb{R} \times \mathbb{R}^n \mapsto \mathbb{R}^n)$, and let $\eta(\cdot) \in \mathrm{C}^1(\mathbb{R} \mapsto \mathbb{R}_{++})$ be a monotone decreasing (resp. increasing) on $\mathbb{R}_+$ (resp. on $\mathbb{R}_-$) function satisfying $\lim_{x\to\pm\infty} \eta(x) = 0$. Consider a singular nonlinear boundary value problem (SNBVP)

$$y' = A(x)y + f(x,y), \quad \|y(x)\| = O(\eta(x)), \quad x \to \pm\infty. \tag{1}$$

Let $Y(\cdot) : \mathbb{R} \mapsto \mathrm{Aut}(\mathbb{R}^n)$ be a solution of the Cauchy problem $Y(0) = \mathrm{Id}$ for the linear system $Y' = A(x)Y$. In what follows, we assume that the linear system admits a *nonuniform strong exponential dichotomy* with positive constants C, α, ε, meaning that there is a decomposition $\mathbb{R}^n = \mathbb{V}_- \oplus \mathbb{V}_+$ into linear subspaces such that $\left\|Y(x)P_\pm Y^{-1}(s)\right\| \le C\mathrm{e}^{\mp\alpha(x-s)+\varepsilon|s|}$ for all $\pm s \le \pm x$, where $P_\pm : \mathbb{R}^n \mapsto \mathbb{V}_\pm$ are projections, $P_+ + P_- = \mathrm{Id}$.

Impose the following additional conditions: (i) $\liminf_{x\to\infty} \frac{\eta'(x)}{\eta(x)} > -\alpha$, $\limsup_{x\to-\infty} \frac{\eta'(x)}{\eta(x)} < \alpha$; (ii) there exists a monotone increasing function $F(\cdot) \in \mathrm{C}(\mathbb{R}_+ \mapsto \mathbb{R}_+)$ such that $\sup_{x\in\mathbb{R}, \|y\|\le r} \frac{\mathrm{e}^{\varepsilon|x|}}{\eta(x)} \|f(x, \eta(x)y)\| \le F(r)$ for all $r \ge 0$; (iii) $\sup_{x\in\mathbb{R}} \|\eta(x)A(x)\| < \infty$. Observe that under the condition (i) one can correctly define the constant

$$K := \sup_{x\in\mathbb{R}} \frac{C}{\eta(x)} \int_{-\infty}^{\infty} \mathrm{e}^{-\alpha|x-s|}\eta(s)\mathrm{d}s.$$

Our main result is as follows:

Theorem. *Suppose that conditions (i) - (iii) are satisfied and let there exists a positive solution r_* of the equation $KF(r) = r$. Then the SNBVP (1) has at least one solution $y(\cdot) \in \mathrm{C}^1(\mathbb{R} \mapsto \mathbb{R}^n)$ such that $\|y(x)\| \le r_*\eta(x)$ for all $x \in \mathbb{R}$.*

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ON THE CAUCHY PROBLEM FOR A ULTRAPARABOLIC EQUATIONS WITH INFINITELY INCREASING COEFFICIENTS IN THE GROUP OF LOWEST TERMS AND STRONGLY DEGENERATIONS IN THE INITIAL HYPERPLANE

We consider an ultraparabolic equation with lowest coefficients infinitely increasing as $x \to \infty$ and degenerations for $t = 0$

$$\alpha(t)\partial_t u - \beta(t)\Bigg(\sum_{j=1}^{n_2} x_{1j}\partial_{x_{2j}} u + \sum_{j=1}^{n_3} x_{2j}\partial_{x_{3j}} u + \sum_{j,s=1}^{n_1} a_{js}\partial_{x_{1j}}\partial_{x_{1s}} u +$$
$$+ b\sum_{j=1}^{n_1} \partial_{x_{1j}}\Big(x_{1j} u\Big)\Bigg) - au = 0, \quad (t,x) \in \Pi_{(0,T]},$$

where n_1, n_2 , n_3 , and n be natural numbers such that $n_3 \le n_2 \le n_1$ and $n = n_1 + n_2 + n_3$; a_{js} , b, and a are real constants, $a_{js} = a_{sj}$, $\{j, s\} \subset \{1, ..., n1\}$, and the following parabolicity.

In [1] we find the fundamental solution G of this equation in the explicit form, establish the properties of the function G and, in particular, deduce exact estimates for G and its derivatives.

By using the function G, we prove the theorems on the integral representations of solutions of the inhomogeneous equation in special L_p-space where degeneration is strong $\int\limits_0^T \frac{dt}{\alpha(t)} = \infty$.

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A CLASS OF SOLUTIONS FOR THE MAXWELL EQUATIONS IN THE KERR SPACETIME AND THEIR PHYSICAL CONSEQUENCES

We have found for the first time in analytic form an exact general solution and solution with separated variables for a class of Maxwell field on Kerr space-time background and have investigated some of their properties and physical consequences. We distinguish this class of Maxwell fields by the condition that they are algebraically special or null, outgoing in Chandrasekhar sense. The general solution generalizes the known solutions of other authors in Minkowski space-time. From the solution with separated variables, we deduce new physical effects: the suppression of right-polarized electromagnetic waves and rotation of plane of polarization, which differs for each harmonics as well as expression for the phase shift.

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APPROXIMATIONS FOR A SOLUTION OF THE NON-STATIONARY EQUATION WHICH DESCRIBES THE NON-LINEAR ACCELERATION OF PARTICLES

We consider the equation for the distribution function $f(t,x,p)$ for particles accelerated at the non-relativistic shock

$$\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right]+\frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p}+Q_{\mathrm{t}}(t)\delta(p-p_{\mathrm{i}})\delta(x) \tag{1}$$

where u is the flow velocity, D the diffusion coefficient, Q_{t} represents the injection at momentum p_{i}. Non-linearity consists in the mutual dependence of $u(t,x)$ and $f(t,x,p)$. The solution for the stationary problem $(t\to\infty)$ is found in [1].

Let us consider that the velocity profile corresponds to the fully developed non-linearity, i.e. $u(t,x)=u(t=\infty,x)$. The direct and inverse Laplace transforms [2] yield us the solution at the shock, i.e. at $x=0$:

$$f_{\mathrm{o}}(t,p)=\mathrm{f}_{\mathrm{o}}(p)\int_0^t Q_{\mathrm{t}}(t-t')\varphi_{\mathrm{o}}(t',p)dt' \tag{2}$$

where $\mathrm{f}_{\mathrm{o}}(p)$ is the non-linear solution of the stationary task [1], $\varphi_{\mathrm{o}}(t)$ is the inverse Laplace transform of

$$\exp\left[-\frac{3}{2}\int_{p_{\mathrm{i}}}^{p}\frac{u_{\mathrm{p}}F_{\mathrm{p}}(s,p')+u_2F_2(s,p')}{u_{\mathrm{p}}-u_2}\frac{dp'}{p'}\right] \tag{3}$$

where the index "p" corresponds approximately to the local values at x reachable by particles with momenum p, index "2" represents the post-shock values. The expression for F_2 is known, e.g. [2]. In our talk, we consider few approximations in order to approach $\varphi_{\mathrm{o}}(t)$.

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CONSEQUENCES OF THE MATHISSON-PAPAPETROU EQUATIONS IN COMOVING TETRADS REPRESENTATION

The Mathisson-Papapetrou equations in terms of the local tetrad coordinates for Schwarzschild's metric are considered. In the case when a spinning particle is moving in the equatorial plane and its spin is orthogonal to this plane, for the particle acceleration $a_{(i)}$ relative to the geodesic free fall from the point of view of the comoving observer we have the equations [1]

$$ma_{(2)} + S_{(1)}(R_{(2)(4)(2)(3)} - \dot{a}_{(3)} - a_{(2)}\gamma_{(2)(3)(4)}) = 0, \tag{1}$$

$$ma_{(3)} + S_{(1)}(R_{(3)(4)(2)(3)} + \dot{a}_{(2)} - a_{(3)}\gamma_{(2)(3)(4)}) = 0 \tag{2}$$

(the first and the second local spatial vectors are orientated along the spin and the direction of the particle motion, respectively, then $a_{(1)} = 0$, $S_{(2)} = 0$, $S_{(3)} = 0$). Using the explicit expressions for the components of the comoving tetrads we calculated the value $\gamma_{(2)(3)(4)}$ of Ricci's coefficient of rotation:

$$\gamma_{(2)(3)(4)} = \frac{u^4(E - mu_4)}{S_0(u_4u^4 - 1)}, \tag{3}$$

where E, m, and S_0 are the particle's energy, mass, and the absolute value of its spin, respectively; u^4 is the component of the particle 4-velocity. The explicit solution of equations (1)–(2) is obtained, where $a_{(2)}(s)$ and $a_{(3)}(s)$ are expressed through $R_{(2)(4)(2)(3)}$, $R_{(3)(4)(2)(3)}$, their derivatives respect to the particle proper time s, and E, m, S_0.

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RECOVERY OF CONTINUOUS FUNCTIONS FROM THEIR NOISY FOURIER COEFFICIENTS

We study the problem of an optimal recovery of functions of one and two variables from their Fourier coefficients with respect to certain orthogonal system that are blurred by noise. The considered function classes consist of continuous, real-valued functions that are given in terms of generalized smoothness. In a more general situation this problem for the classes of smooth and analytic functions defined on various compact manifolds was studied in the classical paper [1].

In the talk we present the results that complement and extend those obtained by Mathe P., Pereverzev S.V. in [2] and Sharipov K. in [3] to the bigger range of function classes and for more general restrictions on the noise level.

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A NOTE ON SEMISCALAR EQUIVALENCE OF POLYNOMIAL MATRICES

Let $\mathbb{F}$ be an infinite field. Denote by $\mathbb{F}_n[\lambda]$ the set of $n \times n$ matrices over the polynomial ring $\mathbb{F}[\lambda]$.

Matrices $A(\lambda), B(\lambda) \in \mathbb{F}_n[\lambda]$ are said to be semi-scalar equivalent if there exist matrices $P \in GL(n, \mathbb{F})$ and $Q(\lambda) \in GL(n, \mathbb{F}[\lambda])$ such that $A(\lambda) = PB(\lambda)Q(\lambda)$ (see [1], Chapter 4). We refer to [2] for the definition of the complexity of semi-scalar equivalence of polynomial matrices. The purpose of this report is to give a criterion of semi-scalar equivalence of two polynomial (2×2) matrices over an infinite field.

Let nonsingular matrices $A(\lambda), B(\lambda) \in \mathbb{F}_2[\lambda]$ be equivalent and $S(\lambda) = \operatorname{diag}(s_1(\lambda), s_2(\lambda))$ be their Smith normal form. For $A(\lambda)$ and $B(\lambda)$ there exist matrices $P_1, P_2 \in GL(2, \mathbb{F})$ and $Q_1(\lambda), Q_2(\lambda) \in GL(2, \mathbb{F}[\lambda])$ such that

$$P_1 A(\lambda) Q_1(\lambda) = s_1(\lambda) \begin{bmatrix} 1 & 0 \\ a(\lambda) & s(\lambda) \end{bmatrix}$$

and

$$P_2 B(\lambda) Q_2(\lambda) = s_1(\lambda) \begin{bmatrix} 1 & 0 \\ b(\lambda) & s(\lambda) \end{bmatrix},$$

where $s(\lambda) = \frac{s_2(\lambda)}{s_1(\lambda)}$ and $\{\deg a(\lambda), \deg b(\lambda)\} < \deg s(\lambda)$ (see [2]). Put $a(\lambda)b(\lambda) = s(\lambda)q(\lambda) + c(\lambda)$, where $\deg c(\lambda) < \deg s(\lambda)$.

Theorem. *Nonsingular matrices $A(\lambda), B(\lambda) \in \mathbb{F}_2[\lambda]$ are semi-scalar equivalent if and only if there exist linear independent vectors $X_1 = \begin{bmatrix} x_{11} & x_{12} \end{bmatrix}$ and $X_2 = \begin{bmatrix} x_{21} & x_{22} \end{bmatrix}$ over $\mathbb{F}$ such that*

$$x_{11}a(\lambda) + x_{12}c(\lambda) + x_{21} + x_{22}b(\lambda) = 0.$$

From the Theorem it follows that the semi-scalar equivalence of polynomial matrices of second order can be decided trivial.

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DETERMINING OF MINOR COEFFICIENT IN SEMILINEAR EIDELMAN TYPE EQUATION

In the domain $Q_T = \mathcal{D}_x \times \mathcal{D}_y \times (0,T)$ we obtained the sufficient conditions of the existence and the uniqueness of a pair of functions $(u(z,t),\ c(t))$, that satisfy the equation, initial, boundary and overdetermination conditions

$$u_t + \sum_{i,j=1}^{k} (a_{ij}(z,t)u_{x_i x_j})_{x_i x_j} - \sum_{i,j=1}^{n} (b_{ij}(z,t)u_{z_i})_{z_j} + \\ +(c(t)+q(z))u + g(z,t,u) = f(z,t), \tag{1}$$

$$u(z,0) = u_0(z), \quad z \in \Omega, \qquad u|_{S_T} = 0, \quad \left.\frac{\partial u}{\partial \nu}\right|_{\partial\mathcal{D}_x \times \mathcal{D}_y \times (0,T)} = 0, \tag{2}$$

$$\int\limits_{\Omega} K(z)u(z,t)\,dz = E(t), \qquad\qquad t \in [0,T]. \tag{3}$$

Here $\mathcal{D}_x \subset \mathbb{R}^k$ and $\mathcal{D}_y \subset \mathbb{R}^l$ are bounded domains with boundaries $\partial\mathcal{D}_x, \partial\mathcal{D}_y \in C^1$; $k,l \in \mathbb{N}$; $\Omega = \mathcal{D}_x \times \mathcal{D}_y$, $S_T = \partial\Omega \times (0,T)$, $x \in \mathcal{D}_x$, $y \in \mathcal{D}_y$, $z = (x,y) \in \Omega$, $n = k + l$, ν is a unit external normal vector to the boundary $\partial\mathcal{D}_x \times \mathcal{D}_y \times (0,T)$, $g(z,t,u)$ is a Lipschitz continuous function with respect to u.

With the use of Faedo-Galerkin method and the method of successive approximations we received the solution such that $c \in C([0,T])$, $u \in L^2(0,T;V_1(\Omega)) \cap C([0,T];L^2(\Omega))$, $u_t \in L^2(Q_T)$, where $V_1(\Omega) = \left\{u : u \in W_0^{1,2}(\Omega), u_{x_i x_j} \in L^2(\Omega), i,j \in \{1,\dots,k\}, \frac{\partial u}{\partial \nu}\big|_{\partial\mathcal{D}_x \times \mathcal{D}_y} = 0\right\}$.

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ON LINEAR HOMEOMORPHISMS BETWEEN FUNCTION SPACES AND THEIR SUBSPACES

For a Tychonoff space X we denote by $C_p(X)$ the space of continuous real valued functions on X, $L_p(X)$ — a space dual to $C_p(X)$. Recall that topological spaces X and Y are called l-equvalent ($X \overset{l}{\sim} Y$) if linear topological spaces $C_p(X)$ and $C_p(Y)$ are linear homeomorphic. Let $X_1 \subseteq X$, $Y_1 \subseteq Y$. We say that the pair (X, X_1) is l-equivalent to the pair (Y, Y_1) ($(X, X_1) \overset{l}{\sim} (Y, Y_1)$) if there exists an linear topological homemorphism $i\colon L_p(X) \to L_p(Y)$ such that $i(\langle X_1 \rangle) = \langle Y_1 \rangle$. Subspace $Y \subseteq X$ is l-embedded in X if there exists linear continuous mapping $\phi\colon C_p(Y) \to C_p(X)$ such that $\phi(f)|_Y = f$ for all $f \in C_p(Y)$.

Theorem. *Let X and Y be a Tychonoff spaces, $X_1 \subseteq X$, $Y_1 \subseteq Y$ — their l-embedded subspaces. Then the following are equivalent:*

1) $(X, X_1) \overset{l}{\sim} (Y, Y_1)$;

2) $X_1 \overset{l}{\sim} Y_1$, $(X/X_1) \overset{l}{\sim} (Y/Y_1)$.

Using the results from [1] we can obtain a complete l-classification of the pairs of the locally compact zero-dimensional metrizable separable spaces and their closed subspaces.

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SH-WAVES SCATTERING BY A PERIODIC ARRAY OF PIEZOELECTRIC INCLUSIONS

The problem of interaction of plane time-harmonic SH-waves with a periodic array of piezoelectric inclusions in an elastic medium is considered. Piezoelectric inclusions have the same shape where the relation of the thickness to the length is a small value. The piezoelectric material is noncontrast and belongs to the 6mm hexagonal class of symmetry. Conditions of the ideal mechanical contact and electric insulation are prescribed on the interface of each inhomogeneity. Asymptotically exact boundary conditions for discontinuities of displacements and their derivatives on the middle line of the central inhomogeneity are obtained using the singular perturbations approach [1]. Models of thin inhomogeneities have been utilized to determine stress-strain state in composites with piezoelectric properties under the dynamic or static loading [2, 3]. The corresponding boundary value problem for discontinuities of displacements and stresses is formulated and solved applying Fourier transformation method. Frequency dependencies of reflection and transmission coefficients are studied for different lattice sizes and electromechanical coupling coefficient values.

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ON LOCAL NEARRINGS OF ORDER p^3

Let R be an additively written non-Abelian group of order p^3 with generators a, b and c satisfying the relations $a^p = 1$, $b^p = 1$, $c^p = 1$, $ab = bac$, $ac = ca$, $bc = cb$. Each element $x \in R$ is uniquely written in the form $x = ax_1 + bx_2 + cx_3$ with coefficients $0 \le x_1 < p$, $0 \le x_2 < p$ and $0 \le x_3 < p$. Moreover, for each $x \in R$ there exist coefficients $\beta(x)$ and $\gamma(x)$ such that $xb = b\beta(x) + c\gamma(x)$. It is clear that they are uniquely defined modulo p, so that some mappings $\beta : R \to Z_p$ and $\gamma : R \to Z_p$ are determined.

Lemma. Let $x = ax_1 + bx_2 + cx_3$ and $y = ay_1 + by_2 + cy_3$ be elements of R. Then the multiplication

$$x \cdot y = ax_1y_1 + b(x_2y_1 + \beta(x)y_2) +$$
$$+c(-x_1x_2\binom{y_1}{2} + x_3y_1 + \gamma(x)y_2 + x_1\beta(x)y_3)$$

with one of the following functions is a nearring multiplication:

- $\beta(x) = x_1^2$ and $\gamma(x) = 2x_1x_2$
- $\beta(x) = 1$ and $\gamma(x) = 0$
- $\beta(x) = x_1$ and $\gamma(x) = 0$
- $\beta(x) = x_1^2$ and $\gamma(x) = 0$
- ...
- $\beta(x) = x_1^{p-1}$ and $\gamma(x) = 0$

Theorem. *There exist at least $p + 1$ non-isomorphic local nearrings on non-metacyclic non-abelian group of order p^3.*

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DIRICHLET-NEUMANN PROBLEM FOR A LINEAR SYSTEM OF HOMOGENEOUS PARTIAL DIFFERENTIAL EQUATIONS

Let's denote: $Q_T = \Omega \times (0,T)$, $T > 0$, Ω^p is a unit circle $\mathbb{R}/2\pi\mathbb{Z}$, $W^m_{\alpha,\beta}$ $(\alpha, \beta \in \mathbb{R})$ is a space of vector functions $\vec{\varphi}(x) = \sum\limits_{k\in\mathbb{Z}} \vec{\varphi}_k e^{ikx}$, $\vec{\varphi}_k = col(\varphi^1_k, \ldots, \varphi^p_k) \in \mathbb{C}^m$, for which is a finite norm

$$\|\vec{\varphi}; W^m_{\alpha,\beta}\| = \sqrt{\sum_{k\in\mathbb{Z}} \sum_{j=1}^{m} |\varphi^j_k|^2 (1+|k|)^{2\alpha} e^{2\beta|k|}}.$$

Let's consider the problem

$$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) \vec{u}(t,x) \equiv \sum_{j=0}^{2n} A_j \frac{\partial^{2n} \vec{u}(t,x)}{\partial t^j \partial x^{2n-j}} = \vec{0}, \quad (t,x) \in Q_T, \qquad (1)$$

$$\left.\frac{\partial^{2j-2} \vec{u}(t,x)}{\partial t^{2j-2}}\right|_{t=0} = \vec{\varphi}_j(x), \left.\frac{\partial^{2j-1} \vec{u}(t,x)}{\partial t^{2j-1}}\right|_{t=T} = \vec{\varphi}_{n+j}(x), j = \overline{1,n}, \qquad (2)$$

where $\vec{u}(t,x) = \mathrm{col}(u^1(t,x), \ldots, u^m(t,x))$, $A_j = \|a^j_{q,r}\|^m_{q,r=1}$, $j = \overline{0,n}$, are square matrices of order m with complex entries, A_n is a identity matrix, $\vec{0} = \mathrm{col}\,(\underbrace{0, \ldots, 0}_{m})$, $\vec{\varphi}_j(x) = \mathrm{col}(\varphi^1_j(x), \ldots, \varphi^m_j(x))$, $j = \overline{1,2n}$.

The conditions under which the problem (1), (2) has a unique solution in the space $C^{2n}([0,T], W^\gamma_{\alpha,\beta})$ are established. One of these conditions is the presence of lower estimations for the sequence of characteristic determinants. The metric approach was applied to establish these estimates [1]. It is proved that such estimations are performed for almost all vectors composed of elements of matrices of system (1) and number $T > 0$.

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GENERAL SOLUTION OF SYSTEMS EQUATIONS IN PARTIAL DERIVATIVES DESCRIBING STRESS STATES FOR ORTHOTROPIC BODIES

Construction of a general solution of the equations theory elasticity of an orthotropic body is an important scientific and practical problem.

Equilibrium equations of orthotropic bodies was used and obtained

$$\frac{\partial}{\partial x_j}\mathrm{L}_{\mathrm{k}}^{1}\mathrm{u}_{\mathrm{k}} = \frac{\partial}{\partial x_k}\mathrm{L}_{\mathrm{j}}^{1}\mathrm{u}_{\mathrm{j}},\ k \neq j,\ k,j=\overline{1,3}, \tag{1}$$

where L_{j}, $\mathrm{L}_{\mathrm{j}}^{1}$ are second-order operators with three variables. In [1], it was shown that if the operators $\mathrm{L}_{\mathrm{j}}^{1}$, $j=\overline{1,3}$ are not equivalent, then the solution of the systems of equations (1) can be represented

$$\mathrm{u}_{\mathrm{j}} = \frac{\partial}{\partial x_j}\prod_{k\neq j}\mathrm{L}_{\mathrm{k}}^{1}\Phi,\ j=\overline{1,3},\ \mathrm{L}_{\mathrm{j}}^{1} = D_{km}L_j - D_{jk}D_{jm}\frac{\partial^2}{\partial x_j^2}, \tag{2}$$

where function Φ satisfies the differential equation of the sixth order.

The missed case [1] was considered when dependence $\mathrm{L}_2^1 = c\mathrm{L}_1^1$, where $c \in R$, will be executed for the entered operators

$$\mathrm{u}_1 = c\frac{\partial\Psi}{\partial x_1} + \mathrm{D}_{23}\frac{\partial\varphi}{\partial x_2}, \mathrm{u}_2 = \frac{\partial\Psi}{\partial x_2} - D_{13}\frac{\partial\varphi}{\partial x_1},$$

$$\mathrm{u}_3 = -\frac{1}{D_{23}}L_4\Psi_1,\ \Psi_1 = \int \Psi \mathrm{dz},$$

where functions Ψ, φ satisfy the equations of the fourth and second order.

It was found that, in contrast to equations that contain only two independent expressions of the derivatives, the obtained equation of the sixth order in three variables does not decompose into factors (second order operators). A spectral algorithm for constructing the solution of these equations for an orthotropic rectangular prism has been developed.

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DIVISIBILITY OF INVARIANT FACTORS OF A MATRIX AND ITS SUBMATRIX

The invariant factors, particular their divisibility, are played an important role in the study problems of arithmetic matrices over different rings [1], [3]. We investigate the divisibility of invariant factors of a matrix and its submatrix over the commutative elementary divisor domains.

Let R be a commutative elementary divisor domain with $1 \neq 0$ [2], $M_n(R)$ be a ring $n \times n$ matrices over R. Consider a nonsingular matrix $A \in M_n(R)$. Since R is an elementary divisor domain there

$$A \sim \mathrm{E} = \mathrm{diag}(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_\mathrm{n}), \quad \varepsilon_\mathrm{i} | \varepsilon_\mathrm{i+1}, \quad \mathrm{i} = 1, \ldots, \mathrm{n} - 1.$$

The matrix E is called the Smith normal form of matrix A, ε_i are invariant factors of matrix A.

The notation $a|b$ means that the element a divides the element b.

Theorem. *Let R be a commutative elementary divisor domain with $1 \neq 0$. Let $A, B, C, D \in M_n(R)$ be a nonsingular matrices and M be a $2n \times 2n$ nonsingular matrix of the form*

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \sim \Gamma = diag(\gamma_1, \ldots, \gamma_{2n});$$

where $\gamma_i | \gamma_{i+1}$, $i = 1, \ldots, 2n - 1$. Then γ_i, $i = 1, \ldots, n$, divides corresponding invariant factors of the matrices A, B, C, D.

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CONJUGATION PROBLEM WITH TWO MULTIPLE NODES FOR SYSTEM OF HYPERBOLIC EQUATIONS OF HIGHER ORDER IN CYLINDRICAL DOMAIN

Let $\Omega^p = (\mathbb{R}/2\pi\mathbb{Z})^p$ be a p-dimensional torus, $\mathcal{D}^p = (-\alpha,\beta)\times\Omega^p$, $\mathcal{D}^p_- = (-\alpha,0)\times\Omega^p$, $\mathcal{D}^p_+ = (0,\beta)\times\Omega_p$, where $p\in\mathbb{N}$, α and β are positive real numbers. The problem we aim to solve is finding a pair of functions $u = u(t,x)$ and $v = v(t,x)$, defined in $\mathcal{D}^p_-$ and $\mathcal{D}^p_+$, respectively, which satisfy the following conditions:

$$\begin{cases} \dfrac{\partial^n u}{\partial t^n} + \sum\limits_{s=1}^{n} a_s \dfrac{\partial^n u}{\partial t^{n-s}\partial x^s} = 0, & (t,x)\in\mathcal{D}^p_-, \\ \dfrac{\partial^m v}{\partial t^m} + \sum\limits_{s=1}^{m} b_s \dfrac{\partial^m v}{\partial t^{m-s}\partial x^s} = 0, & (t,x)\in\mathcal{D}^p_+, \end{cases} \tag{1}$$

$$\frac{\partial^{q-1}u}{\partial t^{q-1}}\Big|_{t=-0} = \frac{\partial^{q-1}v}{\partial t^{q-1}}\Big|_{t=+0}, \quad q=1,\ldots,\theta, \quad x\in\Omega^p, \tag{2}$$

$$\left\{ \begin{array}{lll} \dfrac{\partial^{j-1}u}{\partial t^{j-1}}\Big|_{t=-\alpha} = \psi^\alpha_j(x), & j=1,\ldots,r, & \\ \dfrac{\partial^{j-1}v}{\partial t^{j-1}}\Big|_{t=\beta} = \psi^\beta_j(x), & j=1,\ldots,\ell, & \end{array} \right. \quad x\in\Omega^p, \tag{3}$$

where $a_s, b_s \in\mathbb{R}$, $n,m,r,\ell\in\mathbb{N}$, $0<\theta\le\min\{n,m\}$, $\ell = m+n-\theta-r$, $0<r\le n$, $0<\ell\le m$ and $\psi^\alpha_j, \psi^\beta_j$ are given functions.

In general, this problem are conditionally well-posed and its solvability is related with the problem of small denominators [1]. Using metric approach, we will be discuss the conditions for the solvability of the problem (1)–(3) in Sobolev spaces and the proving estimates for small denominators for almost all (respect to the Lebesgue measure) values of the interpolation nodes α anb β.

1. Ptashnyk B., Il'kiv V., Kmit I., Polishchuk V. Nonlocal boundary value problems for partial differential equations (Kyiv, 2002) (in Ukrainian)

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ORDER ESTIMATES OF THE UNIFORM APPROXIMATIONS BY ZYGMUND SUMS ON THE CLASSES OF CONVOLUTIONS OF PERIODIC FUNCTIONS

We establish the exact-order estimates by the Zygmund sums Z^s_{n-1} (that is trigonometric polynomials $Z^s_{n-1}(f;t) := \frac{a_0}{2} + \sum_{k=1}^{n-1}\left(1-\left(\frac{k}{n}\right)^s\right)\times$ $\times(a_k(f)\cos kt + b_k(f)\sin kt)$, $s>0$, where $a_k(f)$ and $b_k(f)$ are the Fourier coefficients of $f\in L_1$) of 2π-periodic continuous functions f from the classes $C^{\psi}_{\beta,p}$. These classes are defined by the convolutions of functions from the unit ball in the space L_p, $1\le p<\infty$, with generating fixed kernels $\Psi_\beta(t)=\sum_{k=1}^{\infty}\psi(k)\cos\left(kt+\frac{\beta\pi}{2}\right)$, $\Psi_\beta\in L_{p'}$, $\beta\in\mathbb{R}$, $1/p+1/p'=1$. We additionally assume, that the product $\psi(k)k^{s+1/p}$ is general monotone increasing with some power rate, and, besides, for $1<p<\infty$ it holds $\sum_{k=n}^{\infty}\psi^{p'}(k)k^{p'-2}<\infty$, for $p=1$ the inequality $\sum_{k=n}^{\infty}\psi(k)<\infty$ is true.

It is shown that under these conditions Zygmund sums Z^s_{n-1} and Fejer sums $\sigma_{n-1}=Z^1_{n-1}$ realize the order of the best uniform approximations by trigonometric polynomials of those classes, namely for $1<p<\infty$

$$E_n(C^{\psi}_{\beta,p})_C \asymp \mathcal{E}\left(C^{\psi}_{\beta,p}; Z^s_{n-1}\right)_C \asymp \Big(\sum_{k=n}^{\infty}\psi^{p'}(k)k^{p'-2}\Big)^{1/p'},\ 1/p+1/p'=1,$$

$$E_n(C^{\psi}_{\beta,1})_C \asymp \mathcal{E}\left(C^{\psi}_{\beta,1}; Z^s_{n-1}\right)_C \asymp \begin{cases} \sum\limits_{k=n}^{\infty}\psi(k), & \cos\frac{\beta\pi}{2}\ne 0;\\ \psi(n)n, & \cos\frac{\beta\pi}{2}=0,\end{cases}$$

where $E_n(C^{\psi}_{\beta,p})_C := \sup\limits_{f\in C^{\psi}_{\beta,p}}\inf\limits_{t_{n-1}\in\mathcal{T}_{2n-1}}\|f-t_{n-1}\|_C$, $\mathcal{T}_{2n-1}$ is the subspace of trigonometric polynomials t_{n-1} of order $n-1$ with real coefficients, $\mathcal{E}\left(C^{\psi}_{\beta,p}; Z^s_{n-1}\right)_C := \sup\limits_{f\in C^{\psi}_{\beta,p}}\|f-Z^s_{n-1}(t)\|_C$.

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ESTIMATES FOR BEST UNIFORM APPROXIMATIONS OF CLASSES OF CONVOLUTIONS OF PERIODIC FUNCTIONS OF HIGH SMOTHNESS

Denote by $C^{\psi}_{\bar{\beta},p}$, $1 \le p \le \infty$, the set of all 2π-periodic functions f, representable as convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int\limits_{-\pi}^{\pi} \varphi(x-t)\Psi_{\bar{\beta}}(t)dt, \quad a_0 \in \mathbb{R}, \quad \varphi \in B_p^0, \tag{1}$$

$$B_p^0 = \{\varphi \in L_p : \ \|\varphi\|_p \le 1, \ \varphi \perp 1\},$$

with a fixed generated kernel $\Psi_{\bar{\beta}} \in L_{p'}$, $1/p + 1/p' = 1$, the Fourier series of which has the form: $S[\Psi_{\bar{\beta}}](t) = \sum\limits_{k=1}^{\infty} \psi(k)\cos\left(kt - \frac{\beta_k \pi}{2}\right)$, $\beta_k \in \mathbb{R}$, $\psi(k) \ge 0$. A function f in the representation (1) is called $(\psi, \bar{\beta})$-integral of the function φ and is denoted by $\mathcal{J}^{\psi}_{\bar{\beta}}\varphi$ $(f = \mathcal{J}^{\psi}_{\bar{\beta}}\varphi)$. If $\psi(k) \ne 0$, $k \in \mathbb{N}$, then the function φ in the representation (1) is called $(\psi, \bar{\beta})$-derivative of the function f is denoted by $f^{\psi}_{\bar{\beta}}$ $(\varphi = f^{\psi}_{\bar{\beta}})$. The concepts of $(\psi, \bar{\beta})$-integral and $(\psi, \bar{\beta})$-derivative was introduced by A.I. Stepanets.

We find two-sides estimates for the best uniform approximations of classes $C^{\psi}_{\bar{\beta},p}$, $1 \le p < \infty$, of convolutions of 2π-periodic functions from unit ball of the space $L_p, 1 \le p < \infty$, with fixed kernels, modules of Fourier coefficients of which satisfy the condition $\sum\limits_{k=n+1}^{\infty} \psi(k) < \psi(n)$.

In the case of $\sum\limits_{k=n+1}^{\infty} \psi(k) = o(1)\psi(n)$ the obtained estimates become the asymptotic equalities.

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LEBESGUE INEQUALITIES ON THE CLASSES OF GENERALIZED POISSON INTEGRALS IN THE UNIFORM METRIC

Denote by $C^{\alpha,r}_{\beta}C,\ \alpha>0,\ r>0, \beta\in\mathbb{R}$, the set of all 2π–periodic functions, such that for all $x\in\mathbb{R}$ can be represented in the form of convolution $f(x)=\frac{a_0}{2}+\frac{1}{\pi}\int\limits_{-\pi}^{\pi}\varphi(t)\sum\limits_{k=1}^{\infty}e^{-\alpha k^r}\cos\big(k(x-t)-\frac{\beta\pi}{2}\big)dt,\ a_0\in\mathbb{R}, \varphi\perp 1, \varphi\in C.$

The function f in the last equality is called the generalized Poisson integral of the function φ.The function φ is called the generalized derivative of the function f and is denoted by $f^{\alpha,r}_{\beta}$.

By $E_n(f)_C$ we denote the best uniform approximation of the function f by trigonometric polynomials t_{n-1} of the order $n-1$.

For arbitrary $\alpha>0, r\in(0,1)$ we denote by $n_1=n_1(\alpha,r)$ the smallest integer $n\in\mathbb{N}$, such that $\frac{1}{\alpha r}\frac{1}{n^r}\Big(1+\ln\frac{\pi n^{1-r}}{\alpha r}\Big)+\frac{\alpha r}{n^{1-r}}\leq\frac{1}{(3\pi)^3}$.

The following theorem takes place.

Theorem. *Let $\alpha>0,\ r\in(0,1),\ \beta\in\mathbb{R}$ and $n\in\mathbb{N}$. Then, for any function $f\in C^{\alpha,r}_{\beta}C$ and all $n\geq n_1(\alpha,r)$ the following inequality holds*

$$\|f(\cdot)-S_{n-1}(f;\cdot)\|_C\leq e^{-\alpha n^r}\left(\frac{4}{\pi^2}\ln\frac{n^{1-r}}{\alpha r}+\gamma_n\right)E_n(f^{\alpha,r}_{\beta})_C,$$

where $S_{n-1}(f;\cdot)$ are the partial Fourier sums of order $n-1$ of the function f. Moreover, for an arbitrary function $f\in C^{\alpha,r}_{\beta}C$ one can find a function $F(x)=F(f,n,x)$ from the set $C^{\alpha,r}_{\beta}C$, such that $E_n(F^{\alpha,r}_{\beta})_C=E_n(f^{\alpha,r}_{\beta})_C$, such that for $n\geq n_1(\alpha,r)$ the equality holds

$$\|F(\cdot)-S_{n-1}(F;\cdot)\|_C=e^{-\alpha n^r}\left(\frac{4}{\pi^2}\ln\frac{n^{1-r}}{\alpha r}+\gamma_n\right)E_n(f^{\alpha,r}_{\beta})_C.$$

For the quantity $\gamma_n=\gamma_n(\alpha,r,\beta)$ the estimate holds $|\gamma_n|\leq 20\pi^4$.

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ON THE CONSTRUCTION OF SOLUTIONS OF THE EQUATION $f'' + Af = 0$ ACCORDING TO GIVEN SEQUENCES

Let Λ be a sequence of different complex numbers λ_k that does not have finite accumulation points in $\mathbb{C}$ and let M be a sequence of different complex numbers μ_k of multiplicities $q_k \in \mathbb{N}$ that does not have finite accumulation points in $\mathbb{C}$.

Theorem. *For arbitrary sequences* Λ *and* M *of complex numbers,* $\lambda_n \neq \mu_k$, $n, k \in \mathbb{N}$, *there exists a meromorphic function* A *with poles of the second order at the points* λ_k *such that the equation*

$$f'' + Af = 0$$

has a solution f *such that* $f^{1/\alpha}$, $\alpha \in \mathbb{R}\backslash\{0; -1\}$ *is meromorphic function without zeros and with simple poles at the points* λ_k *and the derivative of solution* f' *has zeros of multiplicities* q_k *at the points* μ_k.

Similar results are obtained in [1].

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CANONICAL FORM OF A REDUCED MATRIX WITH ALL NON-ZERO SUB-DIAGONAL ELEMENTS

In this report, we introduce canonical forms for reduced matrices with all non-zero sub-diagonal elements.

Theorem. *Suppose that in the reduced matrix $A(x)$ of the form*

$$A(x) = \begin{Vmatrix} 1 & 0 & 0 \\ a_1(x) & x^{k_1} & 0 \\ a_3(x) & a_2(x) & x^{k_2} \end{Vmatrix}$$

we have $a_1(x), a_2(x), a_3(x) \neq 0$, $a_2(x) = x^{k_1} a_2^{'}(x)$, $q_1 := codeg\ a_1$, $q_2 := codeg\ a_2^{'}$, $q_3 := codeg\ a_3$, $n_j := \begin{cases} q_3, j = 1 \\ q_2 + q_3, j = 3 \end{cases}$, $m_j := q_1 + q_3$, $j = 1, 3$. Then the matrix $A(x)$ is semiscalarly equivalent to the reduced matrix $B(x)$ of the form

$$B(x) = \begin{Vmatrix} 1 & 0 & 0 \\ b_1(x) & x^{k_1} & 0 \\ b_3(x) & b_2(x) & x^{k_2} \end{Vmatrix}$$

where elements $b_1(x), b_2(x), b_3(x) \neq 0$ satisfy one of the following conditions:

1. *$(2q_3)$-monomial is absent in $b_3(x)$, if $q_3 < q_1$ and $q_3 < q_2$;*
2. *$(2q_3)$- and $(q_1 + q_3)$-monomials are absent in $b_3(x)$, if $q_3 < q_1$ and $q_3 > q_2$;*
3. *if $q_3 > q_1$ and $q_3 < q_2$, then in the first of polynomials $b_j(x)$, $j = 1, 3$, which satisfies condition $n_j < k_j$, is absent n_j-monomial, and in the first of these polynomials, which satisfies condition $m_j < k_j$, is absent m_j-monomial;*

The matrix $B(x)$ is uniquely defined.

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IDENTIFICATION OF VAN DER POL OSCILLATOR PARAMETERS

Nonlinear oscillations in many aplications of physics, biology and other sciences are modeled by a van der Pol oscillators. It makes the methods of determining their parameters by an output signals relevant. The problem of the parameters identification for van der Pol equations is considered

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = (\lambda - x_1^2)x_2 - \omega^2 x_1. \tag{1}$$

It is required to find the parameters λ and ω. The known informations are the output $y(t) = (x_1(t), x_2(t))$ and any solution of some additional differential equations

$$\dot{\xi} = U(\xi, x_1(t), x_2(t)), \quad \xi(0) = \xi_0 \in \mathbb{R}^p, \quad p = 2. \tag{2}$$

Problem. *Find asymptotically accurate estimates of the parameters λ and ω from the known values of the output $y(t) = (x_1(t), x_2(t))$ and $\xi(t)$.*

The solution is carried out using the synthesis the invariant relations of a special form for the extended system (1), (2). Such relations determine the unknowns as some functions on known quantities [1]

$$\lambda = \Phi_1(x_1(t), x_2(t)) + \xi_1(t), \ \omega^2 = \Phi_2(x_1(t), x_2(t)) + \xi_2(t). \tag{3}$$

A family of right hands of (2) — functions $U(\xi, x_1, x_2)$ is defined such that for any differentiable $\Phi_i(x_1, x_2), i = 1, 2$ the relations (3) are become invariant for extended system (1), (2). For each such system (2) we have

Main result. If functions $\Phi_1(x_1, x_2) = k\frac{x_2^2}{2}$, $\Phi_2(x_1, x_2) = -kx_1x_2$ and k is positive constant, then

Formulas $\hat{\lambda} = \xi_1(t) + k\frac{x_2^2(t)}{2}$, $\hat{\omega}^2 = \xi_2(t) - kx_1(t)x_2(t)$ determine the asymptotic estimates for the parameter λ and ω^2.

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FUZZY GENERALIZED β-CONTINUITY AND FUZZY GENERALIZED β-γ-CONTINUITY IN FUZZY TOPOLOGICAL SPACES

The aim of this talk is to introduce a new type of fuzzy generalized continuities called fuzzy generalized β-continuity and fuzzy generalized β-γ-continuity. We also formulate the definitions of fuzzy generalized β-open maps, fuzzy generalized β-γ-open maps, fuzzy generalized β-irresolute and fuzzy generalized β-γ-irresolute in fuzzy topological spaces. We discuss the relationship between fuzzy generalized β-continuity and fuzzy generalized β-γ-continuity maps. Furthermore, we demonstrate some of their properties and theorems.

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FOURIER PROBLEM FOR NONLINEAR EVOLUTION SUBDIFFERENTIAL INCLUSIONS

Let $S := (-\infty, 0]$, V be real separable reflexive Banach space with norm $\|\cdot\|$, and H be real Hilbert space with the scalar products $(\cdot,\cdot)$ and norms $|\cdot|$, respectively. Suppose that $V \subset H$ with dense, continuous and compact injection. Denote by V' and H' the dual spaces to V and H, respectively. Identifying H' with subspace of V', and H with H' by the Riesz-Fréchet representation theorem, we have dense and continuous embedding $V \subset H \subset V'$.

Let $\Phi : V \to (-\infty, +\infty]$ be a proper, convex, lower semicontinuous functional, $\Phi(0) = 0$, and $\partial\Phi : V \to 2^{V'}$ be the *subdifferential* of functional Φ with domain $D(\partial\Phi) := \{v \in V \mid \partial\Phi(v) \neq \emptyset\}$.

Additionally, assume that there exist the constants $q > 2$, $K_1 > 0$, $K_2 > 0$ such that

$$(v_1^* - v_2^*, v_1 - v_2) \geqslant K_1|v_1 - v_2|^2 + K_2|v_1 - v_2|^q \quad \forall\, [v_1, v_1^*],\ [v_2, v_2^*] \in \partial\Phi,$$

and for some the constants $p > 2$, $K_3 > 0$ the following holds

$$\Phi(v) \geqslant K_3\|v\|^p \quad \forall\, v \in \mathrm{dom}(\Phi).$$

Assume that $B(t,\cdot) : H \to H$, $t \in S$, is a given family of operators, which satisfy the some conditions.

We consider the evolutionary variational inequality

$$u'(t) + \partial\Phi\big(u(t)\big) + B(t, u(t)) \ni f(t), \quad t \in S, \tag{1}$$

where $f : S \to V'$ is given function.

The *solution* of variational inequality (1) is a function $u : S \to V$ that satisfies the following conditions: 1) $u \in W^1_{p,\mathrm{loc}}(S; V)$; 2) $u(t) \in D(\partial\Phi)$ for a.e. $t \in S$; 3) there exists a function $g \in L^{p'}_{\mathrm{loc}}(S; V')$ such that, for a.e. $t \in S$, $g(t) \in \partial\Phi\big(u(t)\big)$ and $u'(t) + g(t) + B(t, u(t)) = f(t)$ in V'.

This talk is devoted the problem the existence and uniqueness of weak solution of considered problem without conditions on its behavior and growth of data-in at infinity.

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VERBAL WIDTH BY SQUARES OF ALTERNATING GROUP A_N

Verbal width of free group was investigated by Sucharit Sarkar [1]. He proved that an arbitrary commutator of free group of rank greater than 1 can not be generated only by 2 squares. The conditions when a commutator can be presented as a product of 2 squares were found by him too [1].

The commutator subgroup of Sylow 2-subgroups of an alternating group have been previously investigated by the author [2].

We consider a similar problem in the symmetric group S_n. Taking into account that the commutator subgroup of S_n is the alternating group A_n, when $n > 4$ [3], the problem can be reformulated in terms of alternating group.

Therefore, we research verbal width by squares of A_n.

Theorem. *An arbitrary element of A_n can be presented in form of a product of 2 squares of elements from A_n.*

Lemma. *If for an element $g \in A_n$, $\exists l \in \mathbb{N}$, $l = 2k$, $k \in \mathbb{N}$ such that g can be presented in form of a product of independent cycles with odd number of independent l-cycles, then g can not be presented in form of square of h, $h \in A_n$, where h is presented as one cycle.*

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ABOUT APPROXIMATION PROPERTIES OF MULTIPOINT BOUNDARY-VALUE PROBLEMS

We discuss the property of solutions of multipoint boundary-value problems to approximate a solution of the boundary-value problem

$$y^{(r)}(t) + \sum\nolimits_{l=1}^{r} A_{r-l}(t) y^{(r-l)}(t) = f(t) \quad \text{whenever} \quad a \leq t \leq b, \tag{1}$$

$$By = q. \tag{2}$$

Here, $n, r, m \in \mathbb{N}$ and $n \geq r$. We suppose $y \in (C^{(n)})^m := C^{(n)}([a,b], \mathbb{C}^m)$, each $A_{r-l} \in (C^{(n-r)})^{m\times m}$, $f \in (C^{(n-r)})^m$, $q \in \mathbb{C}^{rm}$, and B is a continuous linear operator from $(C^{(n)})^m$ to $\mathbb{C}^{rm}$. We also assume this problem to be uniquely solvable for all f and q.

Let $\mathcal{X}$ be a dense subset of $(C^{(n-r)})^{m\times m}$. Consider a sequence of multipoint boundary-value problems of the form

$$y_k^{(r)}(t) + \sum\nolimits_{l=1}^{r} A_{r-l,k}(t) y_k^{(r-l)}(t) = f(t) \quad \text{whenever} \quad a \leq t \leq b, \tag{3}$$

$$B_k y_k := \sum\nolimits_{j=1}^{p_k} \sum\nolimits_{l=0}^{n} \beta_k^{j,l}\, y_k^{(l)}(t_{k,j}) = q. \tag{4}$$

They are parametrized with $k \in \mathbb{N}$, and their right-hand sides are the same as those of the problem (1), (2). Here, $A_{r-l,k} \in \mathcal{X}$, $p_k \in \mathbb{N}$, $\beta_k^{j,l} \in \mathbb{C}^{rm\times m}$, and $t_{k,j} \in [a,b]$ for all admissible values of k, j, and l.

Theorem. *For the problem* (1), (2), *there exists a sequence of multipoint problems of the form* (3), (4) *such that they are uniquely solvable whenever $k \gg 1$ and that $y_k \to y$ in $(C^{(n)})^m$ as $k \to \infty$. This sequence can be chosen not depending on f and q and can be built explicitly.*
This result was obtained together with H. Masliuk and O. Pelekhata [1].

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ON THE COEFFICIENTS OF TRANSITIVENESS OF THE POSETS MINIMAX ISOMORPHIC TO SUPERCRITICAL POSETS

These studies were carried out together with Prof. V. M. Bondarenko.

Let S be a finite poset and $S^2_< := \{(x, y) \,|\, x, y \in S, x < y\}$. If $(x, y) \in S^2_<$ and there is no z satisfying $x < z < y$, then x and y are called neighboring. Put $n_w = n_w(S) := |S^2_<|$ and denote by $n_e = n_e(S)$ the number of pairs of neighboring elements. The ratio $k_t = k_t(S)$ of the numbers $n_w - n_e$ and n_w ls called the coefficient of transitiveness of S [1]. We assume $k_t = 0$ if $n_w = 0$.

A poset S is called P-critical (resp. NP-critical) if its quadratic Tits form is not positive (resp, is not non-negative), but that of any proper subposet of S is positive (resp. non-negative). In [1] (resp. [2]) we proved that a poset is P-critical (resp. NP-critical) if and only if it is minimax isomorphic to a critical Kleiner poset (resp. a critical Nazarova poset, which also is called a supercritical poset).

We calculated the coefficient of transitiveness for all P-critical posets in [3]. Continuing this topic, we calculate the coefficient of transitiveness for all posets minimax isomorphic to supercritical posets with non-trivial group of automorphisms.

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ON THE METHOD OF FUNDAMENTAL INEQUALITIES FOR THE TWO-DIMENSIONAL CONTINUED FRACTIONS

We describe various analogues of the fundamental inequalities method for the two-dimensional continued fractions. Their applications for investigation of convergence and stability of the two-dimensional continued fractions with complex elements are considered.

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OPTIMAL CONTROL PROBLEM FOR EVOLUTIONARY VARIATIONAL INEQUALITIES WITH VOLTERRA TYPE OPERATORS

Let H be a Hilbert space with the scalar product $(\cdot,\cdot)_H$ and the corresponding norm $\|\cdot\|_H$. We consider the space of controls $U := \{u \in L^2(0,T;H) \mid \int_0^T \omega(t)\|u(t)\|_H^2\,dt < \infty\}$, where $T > 0$ is given, $\omega \in C(0,T)$, and $\omega(t) > 0$ for all $t \in (0,T)$. Let $U_\partial \subset U$ be a convex closed set of admissible controls.

Let $\Phi : H \to (-\infty,+\infty]$ be a proper, convex, lower semicontinuous functional, $\Phi(0) = 0$, and $\partial\Phi : V \to 2^{H'}$ be the *subdifferential* of functional Φ with domain $D(\partial\Phi) := \{v \in H \mid \partial\Phi(v) \neq \emptyset\}$.

Assume that $\mathcal{B} : L^2(0,T;H) \to L^2(0,T;H)$ is a Volterra type operator, which satisfies some additional conditions.

For a given $u \in U_\partial$ the state $y = y(u) = y(\cdot;u)$ of the control evolutionary system is described by initial problem for evolution subdifferential inclusions

$$y'(t) + \partial\Phi\big(y(t)\big) + \mathcal{B}(y)(t) \ni f(t) + Cu(t), \quad t \in (0,T),$$

$$y(0) = y_0,$$

where $C : U \to L^2(0,T;H))$ is a linear continuous operator; $f \in L^2(0,T;H)$; $y_0 \in \mathrm{dom}(\Phi)$.

Under the cost function $J : U \to \mathbb{R}$ we mean

$$J(u) := G(y(u)) + \mu\|u\|_U^2, \quad u \in U,$$

where $G : C([0,T];H) \to \mathbb{R}$ is a lower semi-continuous functional, bounded bellow; $\mu = \mathrm{const} > 0$; $y(u)$ is the state of the control system.

We consider the following **optimal control problem:** find $u^* \in U_\partial$ such that

$$J(u^*) = \inf_{u\in U_\partial} J(u).$$

This talk is devoted to the existence of a solution of optimal control problem under some additional conditions on data-in.

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ONE COMPUTER AUTOMIZED SYSTEM FOR SUPPORT PROCESSES IN THE DIAGNOSTIC DOMAIN

The field of medicine is one of the areas in which the problem of correct real-time information processing is particularly actual. Through the necessity of using modern computational means and, as a result, the inevitability of formalization medical data is naturally justified. The computer automatized system ASPO [1] for support of interrogations and concordance of the primary information in the diagnosis domain has been produced by the author. The intelligent interface [2] that provides diagnostic data, the formation and operation of the system knowledge base, as well as conducting a coordinated examination by an expert group within the computer system is part of the computer system. A set of notions based on predicate calculus is applied to knowledge representation in the field of medical diagnostic in the knowledge base. The collecting diagnostic data about the functioning of a complex object is made in the interactive regime by interviewing people, involved in the diagnosing through electronic hierarchical questionnaires. The questionnaire structure is built automatically during the filling a quest passport. The questionnaires are used for studying the factors forcing on some pathology or group of pathologies and asking the patient at the preliminary diagnosis identification moment.

The functioning of this computer system is tested at the making diagnosis of cardiovascular pathologies and determining the level of the special courses students' knowledge.

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TRANSFORMATION PROPERTIES OF GENERALIZED KAWAHARA EQUATIONS

We study transformation properties of a class of generalized Kawahara equations with time-dependent coefficients of the form

$$u_t + \alpha(t)f(u)u_x + \beta(t)u_{xxx} + \sigma(t)u_{xxxxx} = 0, \quad f_u\alpha\beta\sigma \neq 0, \tag{1}$$

where f, α, β and σ are smooth nonvanishing functions of their variables.

We construct the equivalence groupoid of the class and prove that this class is not normalized but can be presented as a union of two disjoint normalized subclasses singled out from the class (1) by the conditions $f_{uu} \neq 0$ and $f_{uu} = 0$, respectively. Both these subclasses are normalized in the extended generalized sense only.

The criterion of reducibility of variable coefficient equations from the class (1) to their constant coefficient counterparts is derived.

Theorem. *An equation from the class (1) with variable coefficients α, β and σ is reducible to a constant coefficient equation from the same class if and only if the coefficients satisfy the conditions*

$$\left(\frac{\beta}{\alpha}\right)_t = \left(\frac{\sigma}{\alpha}\right)_t = 0, \qquad \textit{for} \quad f_{uu} \neq 0,$$

$$\left(\frac{1}{\alpha}\left(\frac{\beta}{\alpha}\right)_t\right)_t = 0, \quad \left(\frac{\sigma\alpha^2}{\beta^3}\right)_t = 0, \qquad \textit{for} \quad f_{uu} = 0.$$

Using the obtained results and properly gauging the arbitrary elements of the class, we carry out its complete group classification, which covers gaps in the previous works on the subject.

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CONSTRUCTIVE EXISTENCE ANALYSIS OF SOME NON-LINEAR INTEGRAL BVPS

We consider the following non-linear integral boundary value problem

$$\frac{du(t)}{dt} = f\left(t, u(t), \frac{du(t)}{dt}\right), \quad t \in [a, b], \tag{1}$$

$$\int_a^b g(s, u(s), u'(s))ds = d. \tag{2}$$

Here we suppose that the functions f: $[a, b] \times D \times D' \to \mathbb{R}^n$, g: $[a, b] \times D \times D' \to \mathbb{R}^n$ are continuous and satisfy the Lipschitz condition in the domain D and d is a given vector. Let D_a and D_b be a convex subsets of $\mathbb{R}^n$ where one looks for respectively the initial value $x(a)$, and the value $x(b)$ of the solution of the boundary value problem (1), (2).

The problem is to find and establish the existence of a continuously differentiable solution x: $[a, b] \to D$ of the problem (1), (2) with initial value $x(a) \in D_a$.

We note, that the domain D will be defined by using convex linear combinations of subsets D_a and D_b. We introduce the vectors of parameters $z = \mathrm{col}(z_1, ..., z_n)$, $\eta = \mathrm{col}(\eta_1, ..., \eta_n)$ and now, instead of integral problem (1), (2) we will consider the following "model-type" two-point boundary value problem with separated parameterized conditions: $\frac{du(t)}{dt} = f\left(t, u(t), \frac{du(t)}{dt}\right)$, $t \in [a, b]$, $x(a) = z$, $x(b) = \eta$.

We connect the introduced model type problem with the special parameterized sequence of function $x_m(t, z, \eta)_{m=0}^{\infty}$, satisfying the boundary conditions $x(a) = z$, $x(b) = \eta$ for all $z, \eta \in \mathbb{R}^n$. We prove the uniform convergence of the sequence of functions: $x_\infty(t, z, \eta) = \lim_{m\to\infty} x_m(t, z, \eta)$.

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SOME CLASSES OF SYMMETRIC FUNCTIONS ON BANACH SPACES

Let Ω be a Lebesgue measurable subset of the set $\mathbb{R}$ such that $\mu(\Omega) > 0$, where μ is the Lebesgue measure. Let Ξ_Ω be the set of all bijections $\sigma : \Omega \to \Omega$ such that for every Lebesgue measurable set $E \subset \Omega$ sets $\sigma(E)$ and $\sigma^{-1}(E)$ are Lebesgue measurable and $\mu(\sigma(E)) = \mu(\sigma^{-1}(E)) = \mu(E)$. Let $X(\Omega)$ be a vector space of classes of functions on the set Ω, such that for every class $\theta \in X(\Omega)$ and for every bijection $\sigma \in \Xi_\Omega$, the set

$$\theta \circ \sigma := \{x \circ \sigma : \ x \in \theta\}$$

is a class, which belongs to the space $X(\Omega)$.

A function f on the space $X(\Omega)$ is called *symmetric*, if for every $\theta \in X(\Omega)$ and $\sigma \in \Xi_\Omega$,

$$f(\theta \circ \sigma) = f(\theta).$$

Let $n \in \mathbb{N}$. A function f on the nth Cartesian power of the space $X(\Omega)$ is called *symmetric*, if for every $\theta = (\theta_1, \ldots, \theta_n) \in (X(\Omega))^n$ and $\sigma \in \Xi_\Omega$,

$$f(\theta \circ \sigma) = f(\theta),$$

where

$$\theta \circ \sigma = (\theta_1 \circ \sigma, \ldots, \theta_n \circ \sigma).$$

We consider symmetric continuous polynomials, symmetric analytic functions and symmetric continuous $*$-polynomials on real and complex Banach spaces of Lebesgue measurable essentially bounded functions, on real and complex Banach spaces of Lebesgue measurable Lebesgue integrable in a power p $(1 \leq p < +\infty)$ functions and on Cartesian powers of these spaces.

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CLASSIFICATION OF COLORIMETRIC SENSOR DATA ARRAY BY PRINCIPAL COMPONENT ANALYSIS

Human activity inevitably leads to harmful effects on the atmosphere. In order to limit the harmful effects, it is necessary to improve the means of measuring the chemical composition of gaseous media and extremely important to create small effective nanosensors that are sensitive to a wide range of gas mixtures [1, 2]. A key part of measuring instruments is the digital processing of the obtained data, the so-called colorimetric sensor array, which is carried out using modern methods of classification of multidimensional data.

In this research Principal Component Analysis [3] applied for colorimetric sensor data array. The main purposes of this method are reducing the measurability of the data, visualization of the received data on the plane and determining the structure of relationships between variables.

Principal Component Analysis arises from the following optimization problem: for a centered data table $X = [\vec{x}_1^T, \vec{x}_2^T, \dots, \vec{x}_n^T]$ find such a unit vector $\vec{a} = [a_1, a_2, \dots, a_n]^T$, $\vec{a}^T\vec{a} = 1$ which maximize $\sum_{n=1}^{N}(\vec{x}_n^T\vec{a})^2$. Such problem has N solutions $G = [\vec{a}_1, \vec{a}_2, \dots, \vec{a}_N]$ which satisfy matrix equation $S_X\vec{a} = \lambda\vec{a}$, where S_X – covariation matrix of input data X. The transformation G can be considered as a linear transformation of the matrix X. After performing this transformation the variation of the new variables $\mathrm{Var}(Y)$ can be derived from the covariance matrix S_X as a quadratic form, determined by the old variables: $\mathrm{Var}(Y) = G^T S_X G$. Such approach makes it possible to significantly increase the selectivity of the analysis of measuring nanosensor devices.

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NUMERICAL SIMULATION OF PHASE CHANGE TEMPERATURE DISTRIBUTION ON SKIN TISSUE USING EFFECTIVE HEAT CAPACITY AND RADIAL BASIS FUNCTIONS

This article concerns the numerical study of phase change heat distribution in skin tissues during cryosurgery. Here we consider parabolic, single phase lag and dual phase lag bio heat model based on Fourier and Non-Fourier law. A effective heat capacity numerical algorithm is established to convert the non linear governing equation for freezing of biological tissue. The phase change dual phase lag bio heat model is given as

$$\begin{aligned}\tau_q \tilde{C}(T)\frac{\partial^2 T}{\partial t^2} + \left(\tilde{C}(T) + \tau_q c_b \tilde{W}_b(T)\right)\frac{\partial T}{\partial t} \\ = \tilde{K}(T)\left(\frac{\partial^2 T}{\partial x^2} + \tau_t \frac{\partial^3 T}{\partial x^2 \partial t}\right) + c_b \tilde{W}_b(T)(T_a - T) + \tilde{Q}_{met}\end{aligned} \quad (1)$$

We used finite difference approximation and radial basis function approximations for the temporal and spatial variables, respectively to solve the considered model. Relaxation times plays an important role in the study of phase change temperature distribution in skin tissue. We illustrated the graphical representation for effect of thermal relaxation for heat flux (τ_q) and phase lag in temperature gradient (τ_t) on temperature distribution and solidus and liquidus interfaces within the skin tissue.

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THE DERIVED SERIES OF SYLOW 2-SUBGROUPS OF THE ALTERNATING GROUPS AND MINIMAL GENERATING SETS OF THEIR SUBGROUPS

Sylow 2-subgroups of an alternating group have been previously investigated by the author [1]. The commutator width of a group G, denoted by $cw(G)$ [2]. We investigate commutator width [2] of a permutational wreath product $cw(B \wr C_n) \leq \max(1, cw(B))$. Further we prove that the commutator of $Syl_2 A_{2^k}$ has the form $Syl'_2 A_{2^k} = Syl_2 A_{2^{k-1}} \boxtimes Syl_2 A_{2^{k-1}}$, where the subdirect product is defined by k relations. The structure of this commutator for the case $k \neq 2^m$ is additionally found by us. Being more precise, we find that $Syl'_2 A_{4k} = Syl'_2 S_{2^{l_1}} \times Syl'_2 S_{2^{l_2}} \times \cdots \times Syl'_2 S_{2^{l_m}}$. The orders and $Syl_2^{(i)} A_{2^k}$, where $i \in N$ are also found. For an upper bound of $cw(B \wr C_n)$, we also prove that $cw(B \wr C_n) \leq \max(1, cw(B))$, where $n \geq 2$ and B is a group.

Theorem 1. *The order of G''_k is equal to $2^{2^k - 3k + 1}$.*

Lemma. *The index of the third commutator subgroup $Syl_2^{(3)} A_{2^k}$ in $Syl_2 A_{2^k}$ (G_k) is equal to 2^{7k-14}.*

Theorem 2. *The order of third commutator subgroup $Syl_2^{(3)} A_{2^k}$ is equal to $2^{2^k - 7k + 12}$.*

Example *The order of the third commutator subgroup for $k = 64$ is the following* $\left| G_{64}^{(3)} \right| = 2^{64-42+12} = 2^{34}$.

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A STUDY ON GENERALIZED BIVARIATE SUMMATION INTEGRAL TYPE OPERATORS IN POLYNOMIAL WEIGHT SPACE

This article deals with the approximation properties of bivariate summation integral type operators along with their rate of convergence and the convergence property is determined in polynomial weight space. For describing the asymptotic behavior of the proposed operators, the Voronovskaya type theorem is proved. At last, the graphical and numerical representations are given and a comparison of convergence is discussed with bivariate Szász-Mirakjan-Kantorovich operators for the function of two variables.

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ORGANIZATION OF PARALLEL COMPUTING DURING THE RESEARCH OF COMPLEX NETWORK SYSTEMS

To research the state and quality of functioning of complex hierarchical-network systems, a complex methodology be proposed in [1], which is based on preliminary processing of input data, methods of local, forecasting, aggregative and interactive evaluation. This methodology uses large arrays of input data about the objects of the system under research, a significant number of characteristics, parameters and evaluation criteria, takes into account different modes of functioning. Therefore for its implementation it requires significant computational resources and the development of effective parallel algorithms estimation.

For preliminary processing of input data highly efficient parallel algorithms for solving digital filtering problems of different dimension are proposed. A general approach [2] to optimize the methods for the complex evaluation of complex systems using large-block parallelization has been developed. As part of this approach, parallel algorithms [3] for interactive evaluation of systems objects are constructed.

Estimates of the speed up of computations are obtained, which confirm the effectiveness of the proposed parallel algorithms. The program realization of some algorithms on multi-core computers is executed.

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GENERAL SOLUTION OF SECOND-ORDER QUASI-LINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATION UNDER FORM-BOUNDARY CONDITIONS

We study the existence of a general solution of a second-order quasi-linear elliptic partial differential equation under new form-boundary conditions on its coefficients. We will consider a second-order quasi-linear elliptic partial differential equation in the Euclidean space $R^l, \quad l \geq 3$

$$\lambda u - \sum_{i,j=1,\ldots,l} \frac{\partial}{\partial x_i}\left(a_{ij}(x,u)\frac{\partial}{\partial x_j}u\right) + b(x,u,\nabla u) = f, \tag{1}$$

where $f \in L^p \cap L^\infty$ and the a_{ij} is an elliptic matrix. Function $a_{ij}(x,u)$ satisfies the conditions $a_{ij}(x,u)\xi_j - a_{ij}(x,v)\eta_j \geq \mu_6(x)(\xi_i - \eta_i)$, μ_6 is a measurable function and $0 < \delta < \langle \mu_6(\cdot)|u|\rangle < \infty$. Function $b(x,u,\nabla u)$ satisfies the conditions

$$|b(x,u,\nabla u)| \leq \mu_1(x)|\nabla u| + \mu_2(x)|u| + \mu_3(x) \tag{2}$$

$$|b(x,u,\nabla u) - b(x,v,\nabla v)| \leq \mu_4(x)|\nabla(u-v)| + \mu_5(x)|u-v|, \tag{3}$$

where $\mu_1^2 \in PK_\beta, \mu_2 \in PK_\beta, \mu_4^2 \in PK_\beta, \mu_5 \in PK_\beta, \mu_3 \in L^q(R^l)$.

Theorem. *A second-order quasi-linear elliptic partial differential equation* (1) *under the conditions* (2), (3) *and* $\mu_1^2 \in PK_\beta, \quad \mu_2 \in PK_\beta$, $\mu_4^2 \in PK_\beta, \quad \mu_5 \in PK_\beta$, $\mu_3 \in L^q(R^l)$, $f \in L^p \cap L^\infty$, $l > 2$ *and* $\lambda > \lambda_0$ *has a general solution* $u \in W_1^p(R^l, d^l x)$.

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REGULAR BEHAVIOR OF ZERO ORDER ENTIRE FUNCTIONS ALONG CURVES OF REGULAR ROTATION

We set $l_\varphi^\gamma(a,b)=\{z=|z|\exp(i(\varphi+\gamma(|z|)))\}$, $a\le t\le b$; $l_\varphi^\gamma(1,+\infty)=l_\varphi^\gamma$, $\varphi\in\mathbb{R}$, $\gamma(t)$ is a real-valued function differentiated on $[a,b]$. The curve l_φ^γ is called a curve of regular rotation (c.r.r.), if the limit $\lim\limits_{r\to+\infty} r\gamma'(r)=c$, $-\infty<c<+\infty$, exists. Let L be the class of continuously differentiable on $[0,+\infty)$ increasing functions υ such that $r\upsilon'(r)/\upsilon(r)\to 0$ as $r\to+\infty$; $H_0(\upsilon)$ – the class of entire functions of order zero, the counting function $n(r)=n(r,0,f)$ of which zeros satisfies the condition $n(r)=O(\upsilon(r))$, $r\to+\infty$; $n^\gamma(r;\alpha,\beta)$ – number of zeros which lie in the curvilinear sector $D^\gamma(r;\alpha,\beta)=\bigcup\limits_{\alpha<\varphi\le\beta} l_\varphi^\gamma(1,r)$.

We say that zeros of $f\in H_0(\upsilon)$, $\upsilon\in L$, have a υ-density $\Delta^\gamma(\alpha,\beta)$ along c.r.r. l_φ^γ, if for all α, β, $-\pi\le\alpha<\beta<\pi$, except possibly a countable set, the limit $\lim\limits_{r\to+\infty} n^\gamma(r;\alpha,\beta)/\upsilon(r)=\Delta^\gamma(\alpha,\beta)$ exists. The c.r.r. l_θ^γ is ordinary for $f\in H_0(\upsilon)$, if $\lim\limits_{\varepsilon\to 0+}\overline{\lim\limits_{r\to+\infty}}\, n^\gamma(r;\theta-\varepsilon,\theta+\varepsilon)/\upsilon(r)=0$.

Theorem. *Let $\upsilon\in L$, $f\in H_0(\upsilon)$, zeros of f have υ-density $\Delta^\gamma(\alpha,\beta)$ along c.r.r. l_φ^γ. Then a C_0-set E exists such, that for all ordinary c.r.r. l_θ^γ of the function f*

$$\ln f(z)=N^\gamma(r)+iH_f^\gamma(\theta)\upsilon(r)+o\big(\upsilon(r)\big),$$

$z=re^{i\left(\theta+\gamma(r)\right)}\notin E$, $r\to+\infty$, $N^\gamma(r)=N^\gamma(r,0,f)=\int\limits_{l_\theta^\gamma(1,r)} n\big(|w|\big)/w\,dw$,

$H_f^\gamma(\theta)=\int\limits_{\theta-2\pi}^{\theta}(\theta-\psi-\pi)\,d\Delta^\gamma(\psi)$.

The inverse statement holds only if zeros of function f are located on a finite system of curves of regular rotation.

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COMBINATIONAL SECOND HARMONICS OF NONLINEAR LOVE WAVES IN A CRYSTAL LAYER ON A CRYSTAL HALFSPACE

The results of analytic and numeric researches on problem of anharmonic effect analysis of nonlinear wave interaction by several surface shear elastic Love waves propagation are introduced. The nonlinear effects on waves propagation in a structure "monocrystal layer of cubic system m3m class ideally contacting with the analogous class halfspace" are investigated in this problem. The investigations are based on model of general nonlinearity in dynamical deformation process. It lets to use elastic potential with square and cubic deformation components and deformations with nonlinear terms. Approach of nonlinear elastic waves characteristics expansion into series on small parameter is used.

It is shown that anharmonic disturbances for sum of two generalized surface Love waves are introduced by sum of anharmonic disturbances for each of summing waves (considered as monochromatic) and the component that is characterized as combinative type second harmonic. Level of combinative type second harmonic characterizes the nonlinear interaction measure of waves considered.

On the basis of accurate analytic description of homogeneous and inhomogeneous boundary problems that were get using computer algebra method the specific research results of amplitude-frequency characteristics for couple of generalized surface Love waves in considered waveguide structure are get. It is shown that second harmonics of linear surface Love waves are P-SV type waves.

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ON 0-PARAMETER AND 1-PARAMETER SEMIGROUPS OF SPECIAL FORM

These studies were carried out together with Prof. V. M. Bondarenko.

Throughout, K denotes any fixed algebraically closed field. All matrices are considered over K (sometimes with a parameter α running K).

A finitely generated semigroup S is said to be 0-parameter (over K) if there is no matrix representation R_α polynomially depending on a parameter α, such that, for each value $x \in K$ of the parameter, the representation is indecomposable, and for different values of the parameter, the representations are nonequivalent. A semigroup S, not being 0-parameter, is said to be 1-parameter, if the above R_α exists and, moreover, the representations $R_\alpha \otimes J_m(x)$, where m runs the natural numbers and x the field K ($J_m(x)$ denotes the Jordan $m \times m$ block with eigenvalue $x \in K$, $\otimes$ means that α is replaced by $J_m(x)$ and the scalar elements of R_α are "inflated"), cover, up to equivalence, all indecomposable representations of S, except for a finite number in each dimension.

We study matrix representations of oversemigroups of the 0-parameter semigroup $T = \{0, b, c \,|\, b^2 = b, c^2 = c, bc = 0, cb = 0\}$.

Denote, respectively, by (b), (c), (bc), (cb) the defining relations $b^2 = b$, $c^2 = c$, $bc = 0$, $cb = 0$, and introduce the following semigroups:

$T^{(x)} := T \setminus (x) := \{0, b, c \,|\, b^2 = b, c^2 = c, bc = 0, cb = 0 \,\text{without}\, (x)\}$ for $x \in \{(b), (c), (bc), (cb)\}$;

$T^{(x,y)} := T^{(x)} \setminus (y)$ for $x, y \in \{(b), (c), (bc), (cb)\}$, $y \neq x$;

$T^{(x,y,z)} := T^{(x,y)} \setminus (z)$ for $x, y, z \in \{(b), (c), (bc), (cb)\}$, $z \neq x, y$.

Finally, denote by $\mathcal{T}$ the set of all semigroups of the form $T^{(x)}$, $T^{(x,y)}$ and $T^{(x,y,z)}$.

Theorem 1. *A semigroup $S \in \mathcal{T}$ is 0-parameter if and only if $S = T^{(x)}$ for $x \in \{(bc), (cb)\}$.*

Theorem 2. *A semigroup $S \in \mathcal{T}$ is 1-parameter if and only if $S = T^{(x)}$ for $x \in \{(b), (c)\}$ or $S = T^{(x,y)}$ for $x, y \in \{(bc), (cb)\}$.*

AUTHOR INDEX