ТЕОРІЯ ФУНКЦІЙ І ФУНКЦІОНАЛЬНИЙ АНАЛІЗ

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CORRESPONDING CONTINUED FRACTIONS IN MANY VARIABLES

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Correspondence of a continued fraction in many variables to formal multiple power series is the basis of the talk. The correspondence principle is one of the two methods of analytic function representations by many-dimensional continued fractions. Although the main ideas of correspondence go back to Gauss (1813) and Perron (1957), the general theory is described by Jones and Thron (1975, 1979).

The concept of correspondence is also possible to use to define Padé approximants. We consider how the ideas for continued fractions may be generalized to functions of N variables. In 1976-1978 M. O'Donohoe proposed a two-variable generalization of the Stieltjes-type continued fractions and defined them as S_2-fractions, and then this idea was used to get S_N-fractions [2]. The investigation didn't extend so far but can be a foundation for further research. In 1978 Kh. Kuchminska proposed a two-variable generalization of the C-fractions with two orders of correspondence $n+1$ and $2n+1$ [1]. Then in 1980 W. Siemaszko proposed his generalization of the C-fractions [3].

Corresponding definition is being formulated, the structure of proposed corresponding continued fractions, and algorithms of the formal multiple power series expansion into the corresponding fractions with the order of correspondence of its *n*th approximants $n+1$ and $2n+1$ have been constructed.

Let us consider a sequence of rational functions

$$R_k(z) = P_m(z)/Q_l(z),\ z=(z_1,z_2,...,z_N)\in C^N,\ k=1,2,...,$$

$$\deg P_m(z) = m(N),\quad \deg Q_l(z) = l(N).$$

Let us decompose the function $R_k(z)$ into the formal N-multiple power series (FNPS) at $O=(0,...,0)$. It is possible, when $Q_l(0)\neq 0.$ We denote by $T(R_k)$

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http://iapmm.lviv.ua/mpmm2023/materials/ma08_01.pdf the decomposition of $R_k(z)$ into the N-multiple Taylor series at $O=(0,...,0)$.

Definition 1. *The sequence* $\{R_k(z)\}$ *is corresponding to the formal* N*-multiple power series* $P(z)$

$$P(z)=\sum_{|k|=0}^{\infty} c_k z^k,\, z=(z_1,...,z_N)\in C^N,\, k=(k_1,...k_N)\in Z_+^N, \qquad (1)$$
$$|k|=k_1+...+k_N,\;\; z^k=z_1^{k_1}...z_N^{k_N} \;\text{ at } O=(0,...,0),$$

if $\lim_{k\to\infty}\lambda(P-T(R_k(z)))=\infty,\; \lambda(P)=\begin{cases}\infty, & P=0,\\ s, & P\neq 0,\end{cases}\;\; s=|k|.$

Consider an N-dimensional continued fraction (NDCF) corresponding to $P(z)$ in (1)

$$\underset{i=0}{\overset{\infty}{D}}\frac{a_{i,i,...,i}}{\Phi_i(z)}, \qquad (2)$$

where $\Phi_i(z)$ is the sum of p-dimensional continued fractions ($p=1,2,...,N-1$).

Definition 2. *The* N*-dimensional continued fraction (NDCF) is corresponding to (FNPS)* $P(z)$, *if the sequence of its approximants is correspondent to* $P(z)$ *at* $O=(0,...,0)$.

Theorem 1. *Let the* N*-dimensional regular* $C-$*fraction (2) converges uniformly on every compact subset of a certain bounded domain* $D\subset\mathbb{C}$ *(which contains the origin) to an analytic function* $f(z),\ z\in D$.

Then the formal N*-multiple power series (1) is correspondent to the* N*-dimensional regular* $C-$*fraction (2) and also converges in the domain* D *to the function* $f(z)$.

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2. *Murphy J.A., O'Donohoe M.R.* A two-variable generalization of the Stieltjes-type continued fractions // J. Comp. Appl. Math. – 1978. – **4**, № 3. – P. 181–190.
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ВІДПОВІДНІ НЕПЕРЕРВНІ ДРОБИ ВІД БАГАТЬОХ ЗМІННИХ

Побудовано розвинення кратного степеневого ряду у відповідний багатовимірний неперервний дріб, використовуючи як метод Вісковатова, так і його аналоги. Запропоновано теорему про збіжність відповідного ряду.