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SPECTRA OF ALGEBRAS OF ANALYTIC FUNCTIONS, GENERATED BY SEQUENCES OF POLYNOMIALS ON BANACH SPACES AND OPERATIONS ON SPECTRA

Svitlana Vasylyshyn

Vasyl Stefanyk Precarpathian National University

sv.halushchak@gmail.com

Algebras of analytic functions on a Banach space generated by a countable number of polynomials naturally appear as algebras of invariants with respect to actions of groups of symmetry on the Banach space. Typical examples of such algebras are algebras of symmetric analytic functions on ℓ_p, $1 \le p < \infty$, with respect to the group of permutations of the basis vectors or algebras of symmetric analytic functions of bounded type on $L_p(\Omega)$, $1 \le p \le \infty$, with respect to the group of automorphisms of a measure space Ω. The problem of the description of spectra of such kind of algebras is non-trivial ever in the finite dimensional case. For example, the famous Hilbert's fourteenth problem on invariant theory has a negative solution due to the counterexample of Nagata.

Let $\mathbb{I} = \{I_1^{(p)}, I_2^{(p)}, \ldots\}$ be the sequence of n-homogeneous complex-valued polynomials on the space ℓ_p, $1 \le p \le +\infty$, defined as $I_n^{(p)}(x) = x_n^n$ for all $n \in \mathbb{N}$ and $x \in \ell_p$. It is easy to check that the polynomials $I_1^{(p)}, I_2^{(p)}, \ldots$ are continuous and algebraically independent. In this talk, we consider the particular case of countable generated algebras - the algebras generated by the polynomials $\{I_n^{(p)}\}_{n=1}^{\infty}$. We denote by $H_{b\mathbb{I}}(\ell_p)$ the subalgebra of the Fréchet algebra of complex-valued entire functions of bounded type $H_b(\ell_p)$, generated by the set $\mathbb{I}$. We also denote by $M_{b\mathbb{I}} = M_{b\mathbb{I}}(\ell_p)$ the spectrum (the set of all continuous complex-valued linear multiplicative functionals = continuous complex-valued homomorphisms = continuous characters) of $H_{b\mathbb{I}}(\ell_p)$.

In the first part of the talk, we recall some basic notions on the theory of analytic functions on a Banach space.

In the second part of the talk, we provide some information about the spectrum $M_{b\mathbb{I}}(\ell_1)$ of the algebra $H_{b\mathbb{I}}(\ell_1)$.

In the third part of the talk, we consider some shift type operations that can be performed on the set $\tau(M_{b\mathbb{I}}(\ell_p))$, where the mapping $\tau : M_{b\mathbb{I}}(\ell_p) \mapsto \mathbb{C}^\infty$ is defined by $\tau(\varphi) = \{\varphi(I_1^{(p)}), \varphi(I_2^{(p)}), \ldots\}$ for every character $\varphi \in M_{b\mathbb{I}}(\ell_p)$.

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