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WEAKLY SYMMETRIC FUNCTIONS ON SPACES OF LEBESGUE INTEGRABLE FUNCTIONS

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Let X and Y be nonempty sets. Let F be a set of mappings which act from X to itself. A function $f : X \to Y$ is called F-symmetric if $f(a(x)) = f(x)$ for every $a \in F$ and $x \in X$.

Let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a family of sets F_α of mappings which act from X to itself, indexed by elements of some index set Λ, such that for every $\alpha, \beta \in \Lambda$ there exists $\gamma \in \Lambda$ such that $F_\gamma \subset F_\alpha \cap F_\beta$. A function $f : X \to Y$ is called $\mathcal{F}$-weakly symmetric if there exists $\alpha \in \Lambda$ such that f is F_α-symmetric.

Let $L_p[0,1]$, where $p \in [1, +\infty)$, be the complex Banach space of all Lebesgue measurable functions $x : [0,1] \to \mathbb{C}$ for which the pth power of the absolute value is Lebesgue integrable with norm

$$\|x\|_p = \Big(\int_{[0,1]} |x(t)|^p \, dt\Big)^{1/p}.$$

Let $n \in \mathbb{N}$. Let $\Xi^{(n)}_{[0,1]}$ be the set of all bijections $\sigma : [0,1] \to [0,1]$ such that

$$\sigma(t + 1/n) = \sigma(t) + 1/n$$

for every $t \in [0, 1 - 1/n]$ and, for every Lebesgue measurable set $E \subset [0,1]$, both sets $\sigma(E)$ and $\sigma^{-1}(E)$ are Lebesgue measurable and

$$\mu\big(\sigma(E)\big) = \mu\big(\sigma^{-1}(E)\big) = \mu(E),$$

where μ is the Lebesgue measure.

Let $n \in \mathbb{N}$ and $p \in [1, +\infty)$. For $\sigma \in \Xi^{(n)}_{[0,1]}$, let the operator $s_{\sigma,p}$ be defined by

$$s_{\sigma,p} : x \in L_p[0,1] \mapsto x \circ \sigma \in L_p[0,1].$$

Let $S_{n,p} = \big\{s_{\sigma,p} : \ \sigma \in \Xi^{(n)}_{[0,1]}\big\}$ and $\mathcal{S}_p = \big\{S_{2^n,p} : \ n \in \mathbb{N}\big\}$. We shall consider $\mathcal{S}_p$-weakly symmetric functions on $L_p[0,1]$.

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http://iapmm.lviv.ua/mpmm2023/materials/ma08_19.pdf