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NONLOCAL PROBLEM WITH AN INTEGRAL CONDITION FOR A NONHOMOGENEOUS SYSTEM OF EVOLUTION EQUATIONS OF THE SECOND ORDER

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Let H be Banach space, A be a linear operator acting in it, $A: H \to H$, and arbitrary powers for this operator, $A^n, n = 2,3,\ldots$, be also defined in H. Denote $x(\lambda)$ to be an eigenvector of operator A, which corresponds to its eigenvalue $\lambda \in C$.

We consider system of evolution equations

$$\frac{d^2U_i(t)}{dt^2} + \left[\sum_{j=1}^{n} a_{ij}(A)\frac{d}{dt} + b_{ij}(B)\right]U_j(t) = f_i(t) \tag{1}$$

accompanied with the following set of homogeneous nonlocal - integral conditions:

$$\int_{T_1}^{T_2} U_i(t)dt + \int_{T_3}^{T_4} U_i(t)dt = 0, \tag{2}$$

$$\int_{T_1}^{T_2} tU_i(t)dt + \int_{T_3}^{T_4} tU_i(t)dt = 0, \tag{3}$$

where $T_j > 0,\ j = 1,2,3,4,,\ U_i : ([T_1,T_2] \cup [T_3,T_4]) \to H$ is an unknown vector-function, $p_i(\lambda)$, is a given polynomial, $i = 1,2$, $a(A)_{ij}$ and $b_{ij}(B)$ are the abstract operators with the entire symbols $a_{ij}(\lambda) \neq \text{const}, b_{ij}(\lambda) \neq \text{const},\ \lambda \in C$, $f_i(t) : ([T_1,T_2] \cup [T_3,T_4]) \to H$ is a vector–function.

Definition. We shall say that for arbitrary $t \in ([T_1,T_2] \cup [T_3,T_4])$, the vector $f(t)$ from H belongs to $N_F(R,H,\Lambda^*)$, if, on $\Lambda \subseteq C$, there exist a measure $\mu_f(\lambda)$ and analytical in t linear operator $F_f(t,\lambda): H \to H$ such that $f(t)$ can be

http://iapmm.lviv.ua/mpmm2023/materials/ma09_04.pdf represented in the form of Stieltjes integral

$$f(t)=\int_{\Lambda} F_f(t,\lambda)x(\lambda)d\mu(\lambda)\,. \tag{4}$$

Theorem. *Within the context of problem (1) – (3), let* $f(t)$ *belongs to* $N_F(R,H,\Lambda^*)$ *and* $f(t)$ *can be represented in the form (4). Then the formula*

$$U(t)=\int_{\Lambda^*} F_f\left(\frac{d}{dt},\lambda\right)\{P(t,\mu,\lambda)x(\lambda)\}\Big|_{\nu=0} d\mu_f(\lambda)$$

defines a formal solution of the problem (1) – (3), where $P(t,\mu,\lambda)$ *is a solution of equations* $\phi\left(\frac{d}{dt},\lambda\right)P(t,\mu,\lambda)=\exp[\lambda t]$ *and meets the conditions* $\left.\frac{d^kP}{dt^k}\right|_{t=0}=0,$ $k=0,1$.

This result continues the research of work [1, 2, 3].

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НЕЛОКАЛЬНА ЗАДАЧА З ІНТЕГРАЛЬНИМИ УМОВАМИ УМОВАМИ ДЛЯ НЕОДНОРІДНОЇ СИСТЕМИ ЕВОЛЮЦІЙНИХ РІВНЯНЬ ДРУГОГО ПОРЯДКУ

За допомогою диференціально-символьного методу побудовано розв’язок нелокальної задачі з інтегро-диференціальними умовами для системи операторних еволюційних рівнянь другого порядку.