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PROBLEM WITH INTEGRAL CONDITIONS FOR SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS OF THIRD ORDER

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Let K_L be a class of quasi-polynomials of the form $\phi(x)=\sum_{i=1}^{n}Q_i(x)\exp[\alpha_i x]$, where $Q_i(x)$ are given polynomials, $\alpha_i \in L \subseteq R$, $\alpha_l \neq \alpha_k$ for $l \neq k$. Each quasi-polynomial $\phi(x)$ defines a differential operator

$$\phi\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Bigg|_{\lambda=0}=\sum_{i=1}^{n}Q_i\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Bigg|_{\alpha_i}$$

of finite order in the class of certain function $\Phi(\lambda)$.

In the strip $\Omega=\{(t,x)\in R^2 : t\in(T_1,T_2), x\in R\}$, we consider a system of equations

$$\frac{\partial^3 U_i}{\partial t^3}+\sum_{j=1}^{n}a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^2 U_i}{\partial t^2}+\sum_{j=1}^{n}b_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial U_i}{\partial t}+\sum_{j=1}^{n}c_{ij}\left(\frac{\partial}{\partial x}\right)U_j(t,x)=0, \quad (1)$$

$i=1,...,n$, accompanied with the integral conditions

$$\int_{T_1}^{T_2} t^k U_i(t,x)dt=\phi_{ik}(x),\ k=0,1,2, \quad (2)$$

where $a_{ij}\left(\frac{\partial}{\partial x}\right), b_{ij}\left(\frac{\partial}{\partial x}\right), c_{ij}\left(\frac{\partial}{\partial x}\right)$ are the differential expressions with analytical symbols $a_{ij}(\lambda), b_{ij}(\lambda), c_{ij}(\lambda)$. Let $\eta(\lambda)=\int_0^T W^{(n-1)}(t,\lambda)dt$ be a certain function, $W(t,\lambda)$ be a solution of the equation $L\left(\frac{d}{dt},\lambda\right)W(t,\lambda)\equiv 0$, that meets conditions

http://iapmm.lviv.ua/mpmm2023/materials/ma09_05.pdf

$$W^{(n-1)}(t,\lambda)\Big|_{t=0}=1\,,\ \ W^{(n-2)}(t,\lambda)\Big|_{t=0}=0,\ldots,W(t,\lambda)\Big|_{t=0}=0\,.$$

We denote the following:

$$P=\{\lambda\in C:\eta(\lambda)=0\}\,. \tag{3}$$

Theorem. *Let* $\phi_i(x)\in K_M\,, i=1,\ldots,n$ *, then in the class* $K_{M\setminus P}$ *, there exists and a unique solution of the problem (1), (2), where* P *is given in (3), can be represented in the form*

$$U_i(t,x)=\sum_{i=1}^{n}\phi_i\left(\frac{\partial}{\partial\lambda}\right)\left\{\frac{1}{\eta(\lambda)}\,\overset{\sim T}{l}\left(\frac{d}{dt},\lambda\right)W(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\,,$$

where $\overset{\sim T}{l}\left(\frac{d}{dt},\lambda\right)$ *is transpose of a matrix.*

Be means of the differential-symbol method [1], we construction the solution of the problem (1), (2). This work continues results of [2, 3].

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ЗАДАЧА З НЕОДНОРІДНИМИ ІНТЕГРАЛЬНИМИ УМОВАМИ ДЛЯ ОДНОРІДНОЇ СИСТЕМИ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ ІЗ ЧАСТИННИМИ ПОХІДНИМИ

За допомогою диференціально-символьного методу подано розвязок задачі з інтегральними умовами для системи диференціальних рівнянь із частинними похідними. Цей розв’язок існує і єдиний у класі квазімногочленів.