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QUALITATIVE PROPERTIES OF LINEAR AND NONLINEAR FOURTH-ORDER HYPERBOLIC EQUATIONS

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The talk is devoted to the questions of qualitative properties of solutions to hyperbolic fourth-order linear (and quasilinear) equations with constant complex coefficients in the plane bounded domain. The main question is to prove analogue of maximum principle, energetic estimates for initial and boundary value problems, the Goursat problem as a particular case of boundary value problems on characteristics:

$$L(\partial_x)u = a_0\frac{\partial^4 u}{\partial x_1^4} + a_1\frac{\partial^4 u}{\partial x_1^3\partial x_2} + a_2\frac{\partial^4 u}{\partial x_1^2\partial x_2^2} + a_3\frac{\partial^4 u}{\partial x_1\partial x_2^3} + a_4\frac{\partial^4 u}{\partial x_2^4} = f(x). \quad (1)$$

We assume, that Eq. (1) is hyperbolic, that means that all roots of characteristics equation $a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$ are real. More over we will assume that they are different. That means that the angles $\varphi_i - \varphi_j$ between characteristics not equal to zero. We call the angle of characteristics slop the solution to the equation $-\tan\varphi_j = \lambda_j$, where λ_j are real and different roots of the characteristics equation above.

Let us rewrite Eq. (1) in the following form:

$$< \nabla, a^1 >< \nabla, a^2 >< \nabla, a^3 >< \nabla, a^4 > u = f(x)(f(x,u)).$$

Here the vectors $a^j = (a_1^j, a_2^j)$, $j = 1, 2, 3, 4$ are determined by the coefficients a_i, $i = 0, 1, 2, 3, 4$, and $< a, b >= a_1\bar{b}_1 + a_2\bar{b}_2$ is a scalar product.

It is easy to see that vector a^j is a tangent vector of $j-$th characteristic, slope φ_j of which is determined by $-\tan\varphi_j = \lambda_j$, $j = 1, 2, 3, 4$.

In what follows, we also consider the vectors $\tilde{a}^j = (-\bar{a}_2^j, \bar{a}_1^j)$, $j = 1, 2, 3, 4$. It is obvious that $< \tilde{a}^j, a^j >= 0$, so $\tilde{a}^j$ is a normal vector of $j-$th characteristic.

http://iapmm.lviv.ua/mpmm2023/materials/ma09_12.pdf

Let C_j, $j = 1, 2, 3, 4$ are characteristics lines of the Eq. (1). For Eq. (1) we consider several problems in the plane domain $D =: \{(x_1, x_2) : x_1 \in (-\infty, +\infty), x_2 > 0\}$ and obtain local properties of the solution on the arbitrary point $C \in D$.

Let $\Gamma_0 := \{x_1 \in [a, b], x_2 = 0\}$ and consider the Cauchy problem on Γ_0

$$u\,|_{\Gamma_0} = \varphi(x),\ \frac{\partial u}{\partial \nu}\,|_{\Gamma_0} = \psi(x),\ \frac{\partial^2 u}{\partial \nu^2}\,|_{\Gamma_0} = \sigma(x),\ \frac{\partial^3 u}{\partial \nu^3}|_{\Gamma_0} = \xi(x). \tag{2}$$

We define domain Ω as a domain which is restricted by the characteristics C_j, $j = 1, 2, 3, 4$ and Γ_0. Then from Green formula, we have:

$$\int\limits_{\Omega} \{Lu \cdot \bar{v} - u \cdot \overline{L^{+}v}\} dx = \sum_{k=0}^{3} \int\limits_{\partial\Omega} L_{(3-k-1)} u \cdot v_{\nu}^{(k)} ds. \tag{3}$$

Here $L_{(3-k-1)}u$, $k = 0, 1, 2, 3$ are $L-$traces of the solution u, which are defined by the boundary value of u.

For operator (1) in domain Ω restricted by characteristics C_j and Γ_0 Green formula takes the form:

$$\int\limits_{\Omega} Lu \cdot \bar{v}\, dx = \int\limits_{\Omega} < \nabla, a^1 >< \nabla, a^2 >< \nabla, a^3 >< \nabla, a^4 > u \cdot \bar{v}\, dx =$$

$$\int\limits_{\partial\Omega} < \nu, a^1 >< \nabla, a^2 >< \nabla, a^3 >< \nabla, a^4 > u \cdot \bar{v}\, ds -$$

$$\int\limits_{\Omega} < \nabla, a^2 >< \nabla, a^3 >< \nabla, a^4 > u \cdot \overline{< \nabla, a^1 > v}\, dx.$$

Put $v = 1$ and calculate $L_{(3)}u =< \nu, a^1 >< \nabla, a^2 >< \nabla, a^3 >< \nabla, a^4 > u$, $L_{(3)}-$ trace on $\partial\Omega = C_1 \cap C_2 \cap C_3 \cap C_4$.

The main result is the following analog of the maximum principle for the fourth-order hyperbolic equations:

Theorem 1. *Let $u \in H^m(\Omega)$, $m \geq 4$, satisfy the following inequalities:*

$$Lu \geq 0,$$

in Ω,

$$u\,|_{\Gamma_0} \leq 0,\ u_\nu|_{\Gamma_0} \leq 0,\ u_{\nu\nu}|_{\Gamma_0} \leq 0,$$

$$L_{(3)}u|_{\Gamma_0} \leq 0,$$

then $u \leq 0$ *in* Ω.