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EVOLUTION STOKES SYSTEM OF THE THIRD ORDER WITH VARIABLE EXPONENT OF NONLINEARITY

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Let $n \in \mathbb{N}$ and $T > 0$ be fixed numbers, $n \geq 2$, $\Omega \subset \mathbb{R}^n$ be a bounded domain with the smooth boundary $\partial\Omega$, $Q_{0,T} = \Omega \times (0, T)$.

We consider the problem of finding a pair of functions $\{u, \pi\}$ that satisfy the following relations:

$$u_{tt} - \sum_{i,j=1}^n \left(A_{ij}(x,t) u_{tx_i} \right)_{x_j} - \sum_{i,j=1}^n \left(B_{ij}(x,t) u_{x_i} \right)_{x_j} + G(x) |u_t|^{q(x)-2} u_t + \int_{\Omega} \mathfrak{Z}(x,t,y) u_t(y,t) dy + \nabla \pi = f(x,t) \quad \text{in } Q_{0,T}, \quad (1)$$

$$\operatorname{div} u = 0 \quad \text{in } Q_{0,T}, \quad (2)$$

$$\int_{\Omega} \pi(x,t) dx = 0 \quad \text{in } (0, T), \quad (3)$$

$$u|_{\partial\Omega \times [0,T]} = 0, \quad (4)$$

$$u|_{t=0} = u_0(x) \quad \text{in } \Omega, \quad (5)$$

$$u_t|_{t=0} = u_1(x) \quad \text{in } \Omega, \quad (6)$$

where $u = (u_1, \dots, u_n) : Q_{0,T} \rightarrow \mathbb{R}^n$ is the velocity field, $\pi : Q_{0,T} \rightarrow \mathbb{R}$ is the pressure, $\nabla \pi = (\frac{\partial \pi}{\partial x_1}, \dots, \frac{\partial \pi}{\partial x_n})$, $\operatorname{div} u = \frac{\partial u_1}{\partial x_1} + \dots + \frac{\partial u_n}{\partial x_n}$, A_{ij} , B_{ij} ($i, j = \overline{1, n}$), G , \mathfrak{Z} are some matrices, f is some vector function. The function $q(\geq 1)$ is called the variable exponent of nonlinearity to the equation (1).

Under additional conditions for data-in, we prove the solvability of problem (1)–(6).