UDC 517.9

HOPF BIFURCATION FOR GENERAL 1D SEMILINEAR WAVE EQUATIONS WITH DELAY

Iryna Kmit

Humboldt University of Berlin,
Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine

irina.kmit@hu.berlin.de

We consider boundary value problems for 1D autonomous damped and delayed semilinear wave equations of the type

$$\partial_t^2 u(t,x) - a(x,\lambda)^2 \partial_x^2 u(t,x) = b(x,\lambda,u(t,x),u(t-\tau,x),\partial_t u(t,x),\partial_x u(t,x))$$

with Dirichlet-Neumann boundary conditions. We state conditions ensuring Hopf bifurcation, i.e., existence, local uniqueness (up to time shifts), regularity (with respect to t and x) and smooth dependence (on τ and λ) of small non-stationary time-periodic solutions, which bifurcate from the stationary solution $u = 0$, and we derive a formula which determines the bifurcation direction with respect to the bifurcation parameter τ.

To this end, we transform the wave equation into a system of partial integral equations and then apply a Lyapunov-Schmidt procedure and a generalized implicit function theorem. The main technical difficulties, which have to be managed, are typical for hyperbolic PDEs (with or without delay): small divisors and the “loss of derivatives” property.

This is joint work with Lutz Recke.

Kmit I., Recke L. Hopf bifurcation for general 1D semilinear wave equations with delay // J. Dyn. Dif. Eqs. – 2022. – **34**. – P. 1393–1431.

http://iapmm.lviv.ua/mpmm2023/materials/ma09_19.pdf