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OPTIMAL CONTROL OF A HYPERBOLIC SYSTEM OF COUNTABLE SEMILINEAR EQUATIONS

Oleksandr Hnatiuk, Volodymyr Kyrylych, Olha Milchenko

Ivan Franko National University of Lviv

gnatyksasha1997@gmail.com, vkyrylych@ukr.net, olga.milchenko@lnu.edu.ua

In the domain $V = \{x, t : 0 < t < T, -kt < x < kt, k > 0\}$, consider a certain process $\{y, z, v\}(x, t)$, the evolution of which in time and space is described by a degenerate (presence of the orthogonal to the coordinate axes characteristics) hyperbolic system of countable semilinear equations of the first order

$$\begin{cases} \dfrac{\partial y_i}{\partial t} + \lambda_i(x,t)\dfrac{\partial y_i}{\partial x} = f_i(x,t,y_1,y_2,\ldots,z_1,z_2,\ldots,v_1,v_2,\ldots), \\ \dfrac{\partial z_i}{\partial t} = g_i(x,t,y_1,y_2,\ldots,z_1,z_2,\ldots,v_1,v_2,\ldots), \\ \dfrac{\partial v_i}{\partial x} = \rho_i(x,t,y_1,y_2,\ldots,z_1,z_2,\ldots,v_1,v_2,\ldots), i \in I, \end{cases} \tag{1}$$

where $I = \{i : i = 1, 2, \ldots\}$.

Let us supplement the system (1) with boundary conditions

$$\begin{aligned} y_i(-kt,t) &= \gamma_i^1(t, y_s(-kt,t)_{s\in I_1}, u^{(1)}(t)), i \in I_2 \cup I_3, \\ y_i(kt,t) &= \gamma_i^2(t, y_s(kt,t)_{s\in I_2}, u^{(2)}(t)), i \in I_1 \cup I_3, \\ z_i(-kt,t) &= \gamma_i^3(t, u^{(3)}(t)), i \in I, \\ z_i(kt,t) &= \gamma_i^4(t, u^{(4)}(t)), i \in I, \\ v_i(-kt,t) &= \gamma_i^5(t, u^{(5)}(t)), i \in I. \end{aligned} \tag{2}$$

Here I_1, I_2, I_3 are the sets of indices defined as follows:

$$I_1 = \{i \in I,\ \lambda_i(0,0) < -k\},\ I_2 = \{i \in I,\ \lambda_i(0,0) > k\},$$
$$I_3 = \{i \in I,\ -k < \lambda_i(0,0) < k\}.$$

These sets can be empty, consisting of a finite or countable number of elements. Let them contain s_1, s_2, s_3 elements, respectively. Without limiting generality, we can assume that s_1, s_2 and s_3 are equal to $\aleph_0$. We denote by $u^{(r)} (r = \overline{1,5})$

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the control vector functions such that for compacts $U^r(r=\overline{1,5})$, $u^{(r)}:\mathbb{R}_+\to U^r\subset\mathbb{R}^{r_0}$, $(r_0\in\aleph_0)$.

Let the target function for $y=(y_1,y_2,\ldots)$, $z=(z_1,z_2,\ldots)$, $v=(v_1,v_2,\ldots)$ be

$$J(u^{(1)},\ldots,u^{(5)})=\int\limits_0^T G_0(y_j(-kt,t)_{j\in I_1},y_j(kt,t)_{j\in I_2},z_j(-kt,t)_{j\in I},$$

$$z_j(kt,t)_{j\in I},v_j(-kt,t)_{j\in I},t)dt+\iint\limits_V G(y,z,v,t)dxdt. \quad (3)$$

We need to study the problem

$$\min_{\nu_{ad}} J(u^{(1)},\ldots,u^{(5)}), \quad (4)$$

where by ν_{ad} we mean the set of admissible sets $\{y,z,v,u^{(1)},u^{(2)},u^{(3)},u^{(4)},u^{(5)}\}$, for which, when choosing the controls $\{u^{(1)},u^{(2)},u^{(3)},u^{(4)},u^{(5)}\}$, there exists a unique, generalised in a certain sense, solution of the problem (1)-(2) [1].

The peculiarities of problems for countable systems of ordinary differential equations and their applications are described in [4], the methodology for solving such problems for hyperbolic systems are given in [1, 3], and the problems of optimal control of hyperbolic systems are considered in [2].

For the proposed variant of the optimal control problem, the theorem of fulfilling the necessary conditions of optimality is proved.

The variational analysis of the problem under study is based on the use of variations that ensure smoothness of the admissible controls.

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ОПТИМАЛЬНЕ КЕРУВАННЯ ГІПЕРБОЛІЧНОЮ СИСТЕМОЮ ЗЛІЧЕННИХ НАПІВЛІНІЙНИХ РІВНЯНЬ

Отримано необхідні умови оптимальності задачі оптимального керування виродженою гіперболічною системою зліченних напівлінійних рівнянь.