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COMPLETE REPRESENTABILITY OF SOCIALLY DISTANT SETS

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As it happens with calamities, the COVID-19 pandemic suggested a mathematical problem. Namely, Segal-Halevi formulated the following generalization of the combinatorial geometry problem from [2]. Given a family of sets of socially distant people, we have to choose a set of socially distant representatives of the family. On the other hand, to ensure inclusion, diversity, and equity, the set of representatives is required to be complete, that is each set of the family provides exactly one representative. Unfortunately, it turns out that similarly to the other result from the theory of social choice, the famous Arrow's theorem [1], even in this idealized simple mathematical model, the fair choice is hard and even impossible in some cases. Therefore there arises the decidability question of the fair choice problem, which remains open.

Now let us speak formally. Let X be a metric space. A subset $A \subseteq X$ is *separated*, if the distance between each two distinct points of A is at least 2. Given a natural number n, a sequence $(a_1, \ldots, a_n)$ of distinct points of X is *separated*, if the set $\{a_1, \ldots, a_n\}$ is separated. A sequence $(k_i)_{i\in[n]} \in \mathbb{N}^n$ is *representable*, if for any sequence $(A_i)_{i\in[n]}$ of separated sets with $|A_i| = k_i$ for each $i \in \{1, \ldots, n\}$, the Cartesian product $\prod_{i=1}^n A_i$ contains a separated sequence.

Our aim is to investigate general properties of representable sequences and to detect the latter for a few simple spaces X, namely, when X is the real line or the plane, both endowed with the Euclidean metric, or the plane endowed with the ℓ_∞ metric.

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2. *Segal-Halevi E.* Choosing representatives with spatial separation // Mathematics Stack Exchange, http://math.stackexchange.com/q/3702472.

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