UDC 517

APPROXIMATION RELATIONS ON THE POSETS OF PSEUDOULTRAMETRICS

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Recall that a poset $(D, \leq)$ is directed (resp., filtered) if for all $d_1, d_2 \in D$ there is $d \in D$ such that $d_1, d_2 \leq d$ (resp., $d_1, d_2 \geq d$).

Definition 1. An element x_0 is called to be way below an element x_1 (or approximates x_1 from below) in a poset $(X, \leq)$ (denoted $x_0 \ll x_1$) if for every non-empty directed subset $D \subset X$ such that $x_1 \leq \sup D$ there is an element $d \in D$ such that $x_0 \leq d$.

Definition 2. An element x_0 is called to be way above an element x_1 (or approximates x_1 from above) in a poset $(X, \leq)$ (denoted $x_0 \gg x_1$) if for every non-empty filtered subset $D \subset X$ such that $x_1 \geq \inf D$ there is an element $d \in D$ such that $x_0 \geq d$.

Obviously $x_0 \ll x_1$ or $x_0 \gg x_1$ imply respectively $x_0 \leq x_1$ or $x_0 \geq x_1$. A poset is called continuous if each element is the least upper bound of the directed set of all elements approximating it from below (resp., the greatest lower bound of the filtered set of all elements approximating it from above). In this paper we are not interested in "way above" relation and adopt the following definition.

Definition 3. An element x_0 is called weakly way below an element x_1 in a poset $(X, \leq)$ (denoted $x_0 \prec\!\!\prec x_1$) if for every non-empty directed subset $D \subset X$ such that $x_1 = \sup D$ there is an element $d \in D$ such that $x_0 \leq d$.

The partial orders on the set $PsU(X)$ of all pseudoultrametrics on X and its subsets $CPsU(X)$ and $LCPsU(X)$ are defined pointwise: a pseudoultrametric d_1 precedes a pseudoultrametric d_2 (written $d_1 \leq d_2$ or $d_2 \geq d_1$) if $d_1(x, y) \leq d_2(x, y)$ holds for all points $x, y \in X$. The trivial pseudometric $d \equiv 0$ is the least element of $PsU(X)$, $CPsU(X)$, and of $LCPsU(X)$. We write $d_1 < d_2$ or $d_2 > d_1$ if $d_1 \leq d_2$ and $d_1 \neq d_2$ (this does not mean that $d_1(x, y) < d_2(x, y)$ for all x, y).

Theorem 1. *Let d, d_0 be pseudoultrametrics on X, $d_0 \notin CPsU(X)$. Then d_0 is not weakly way below d (hence is not way below d) neither in $PsU(X)$ nor in $LCPsU(X)$.*

http://iapmm.lviv.ua/mpmm2023/materials/ma10_23.pdf