REFINED MATHEMATICAL MODEL OF FLUTTER FOR COMPOSITE PLATE-STRIP

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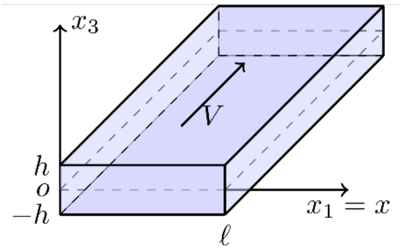
Modern trends toward reducing material intensity while ensuring the necessary level of reliability in critical structures have led to an increased demand for thin-walled structures made of composite materials, particularly those reinforced with polymer matrices. The primary difference between such materials and traditional homogeneous isotropic materials is their susceptibility to transverse shear.

Plate elements are among the essential components of load-bearing structures in the aerospace industry [5]. Their mechanical behavior under operational, especially dynamic, loads often plays a crucial role in maintaining the integrity and functional suitability of these objects. This is especially relevant when the element is in an aerodynamic gas flow (see figure).



At certain flow velocities and with specific geometric and physical-mechanical characteristics of the plate, a process of self-excited vibrations with increasing amplitude may occur, potentially leading to the destruction of both the plate and the entire structure. This phenomenon is commonly known as flutter [2, 4]. Flutter in plates has been sufficiently studied using classical theories [1], which do not account for the specific deformation characteristics of composites. Therefore, it is essential to develop mathematical models of flutter for composite plates that allow for analytical solutions. These models, in addition to their practical value, can be used to verify numerical and experimental methods for studying the flutter of more complex objects.

In the proposed work, a rectangular plate is considered, with one dimension in the mid-plane significantly smaller than the other. Such a thin-walled element is usually called a plate-strip. One-dimensional relations of the refined plate theory [3] were used to model its dynamic deformation. This theory accounts for the material's susceptibility to transverse shear and precisely satisfies the boundary conditions for the forces on the surface planes. Based on this, a solvable equation for the dynamic deflection w is derived:

$$w^{IV} - \frac{2\rho h}{D}\ddot{w} - \frac{2\rho h}{\Lambda}\ddot{w}'' - \frac{M_a}{D}\left(\frac{D}{\Lambda}w''' + \frac{D}{V}\dot{w}'' - w' - \frac{1}{V}\dot{w}\right) = 0\,, \qquad (1)$$

where M_a – is a characteristic of the aerodynamic flow, which depends on the Mach number, which has a velocity V; D and Λ are the bending and shear stiffnesses of the plate-strip, $2h$ is its thickness, and ρ is the material density.

To obtain an analytical solution to equation (1), a characteristic equation (2) for the mode parameters of self-excited vibrations was derived. Unlike the classical case, this equation is a full fourth-order polynomial. Its reduced form was constructed, and based on Ferrari's method an algorithm for solving it was developed and implemented in software. The algorithm was verified using known theoretical and experimental results.

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УТОЧНЕНА МАТЕМАТИЧНА МОДЕЛЬ ФЛАТТЕРА КОМПОЗИТНОЇ ПЛАСТИНИ-СМУГИ

Запропонована уточнена математична модель флаттера композитної пластини-смуги, що дозволяє отримати аналітичний розв'язок.